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Determination of Atmospheric Temperature and Pressure Profiles from Astronomical Refraction Measurements Near the Horizon

N. A. VASILENKO, K. P. GAYKOVICH, AND M. I. SUMIN

Scientific-Research Radiophysics Institute

The results of the reconstruction of the altitude profiles of atmospheric parameters from measurements of the optical refraction of stars from the Earth's surface at positive elevation angles are presented. The reconstruction is based on the solution of the corresponding inverse problem for the refractive index profile. A. N. Tikhonov's method and the method of statistical regularization are used. Comparison of the reconstructed profiles with data from simultaneous meteorological sounding of the atmosphere confirmed the theoretical estimates of the accuracy of the reconstruction and demonstrated the usefulness of the method developed for long-range sounding of the atmosphere. Examples of the reconstruction of temperature profiles with inversions are presented.

INTRODUCTION

In this paper we present the results of the reconstruction of altitude profiles of atmospheric variables from measurements of the optical refraction of stars at small positive elevation angles.

The reconstruction procedure is based entirely on the results of a theoretical analysis of the problem and the algorithms developed in [1, 2]. In these works the problem is formulated in the form of a Fredholm integral equation of the first kind

$$\int_{\rho_0}^{\rho_H} N(\rho)\,\frac{\rho x}{(\rho^2-x^2)^{3/2}}\,d\rho = \tilde{\varepsilon}(x), \qquad (1)$$

where A? is the index of refraction related to the refractive index by the relation *N=10"(n*– 1); *p=nr, p_a = n(r_a)r_{0t}* x=n(r_0)r_0 cos ϑ_0; $\tilde{\mathcal{E}}$ is the right side of the equation related to the measured refraction e by the relation

$$\tilde{\varepsilon}(x) = -10^6\varepsilon(x) + x\left(\frac{N(\rho_0)}{\sqrt{\rho_0^2-x^2}} - \frac{N(\rho_H)}{\sqrt{\rho_H^2-x^2}}\right). \qquad (2)$$

V is the radius vector from the center of the Earth, r_ is the Earth's radius, 6 is the elevation angle, and $p_s=n(r_a)r_H$, where $r_a = r_a + H$. The altitude *H is* very large (see [2]). The distribution *N(p)* obtained by solving (1) is transformed into the altitude profile *N(h)* with the help of the relation *p = nr=n(r_a + h).* In the optical range, knowing *N(h)* it is possible to reconstruct also the altitude profiles of the pressure (P) and temperature *(T),* using the equation of statics and of the gaseous state (see [3]).

In [1, 2] it is shown that to solve Eq. (1) which is a classical example of an improperly posed problem, quite significant *a priori* information about the profile sought *N(p)* must be used. Two algorithms for reconstructing *N(p)* were developed based on the well-known methods for solving problems of this type. The first one is based on the fact that the exact solution *N(p)* belongs to a compact set of monotonically nonicreasing functions [4], while the second one is based on the use of the covariational interlevel couplings of *N(p)* [1] (the method of statistical regularization).

The analysis of the informative range of angles, in which the refraction must be measured, carried out in [2] was used in designing the experiment. It is well known that as the elevation angle increases, the refraction is determined with increasing accuracy by the value of the refractive index at the ground (the so-called "Laplace theorem"). Thus, variations of the refraction owing changes in the stratification of the atmosphere decrease rapidly as the elevation angle increases. For a fixed measurement accuracy this effect gives an upper limit to the informative range of angles. It was established based on a statistical analysis (carried out for large climatic ensembles of *N(p)*, obtained from aerological sounding data) that for the accuracy currently achievable in measurements of refraction in the optical range (1 - 10")the range of informative angles is <4°.

The solution of the inverse problem and the reconstruction of the profiles of atmospheric parameters were modeled numerically both by the method based on the property of monotonicity of *N(p)* and by the method of statistical regularization. Estimates of the errors in the reconstruction as a function of the level of the modeled

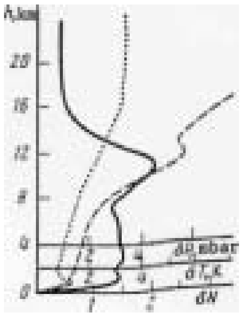

Fig. 1. Rms error in the reconstruction of the atmospheric parameters (index of refraction (solid line), temperature (broken line), pressure (dots)). Summer ensemble for the European Territory of the USSR, $\delta\varepsilon = 5''$.

measurement error, were obtained based on the statistical analysis of results for large ensembles, corresponding to winter and summer conditions in the central part of the European Territory of the USSR. Figure 1 shows the rms errors in the reconstruction of the atmospheric parameters by the method of statistical regularization as a function of the altitude for a measurement accuracy of $\delta\varepsilon = 5''$, close to that realized experimentally (for the summer ensemble). Up to an altitude of ~*6* km the index of refraction is determined with accuracy $\delta N < 1.5$, the pressure with accuracy $\delta P < 1$ mbar, and the temperature with accuracy $\delta T<1.5$ K, which is comparable to the best theoretical estimates of the accuracies of reconstruction by methods of ground-based microwave radiometry [6]. It was established that for the achievable measurement accuracies the method of statistical regularization is preferable. It was demonstrated that temperature inversions can be reconstructed. In order to employ the method the fact, established in [2], that the number of measurements of refraction in the informative range of angles has virtually no effect on the accuracy of reconstruction-five to ten measurements are sufficient—is important. Thus, the results of the theoretical analysis of [1, 2] have demonstrated the utility of employing measurements of refraction at small elevation angles for determining the stratification of the atmosphere. At the same time, the theoretical analysis ignored a number of important questions associated with the practical application of the method, answers to which could be obtained only from experimental results. First, there is the question of the role of horizontal nonuniformities of the refractive index in the real atmosphere. Since the theory is constructed in the approximation of a spherically symmetric atmosphere while informative measurements can be made only at small elevation angles, the existence of horizontal nonuniformities introduces an additional error. The effect of horizontal nonuniformities on the magnitude of the refraction under different conditions has not yet been adequately studied, so that it is difficult to evaluate the corresponding errors on the basis of a theoretical analysis. Without experimental results it is also difficult to take into account the errors introduced into the refraction by the turbulent fluctuations of the refractive index, and also the errors associated by the dispersion of the refractive index in the optical range.

It is also necessary to take into account systematically the moisture content, which makes a contribution to the refractive index and also appears in the barometric formula, which is important in calculating the pressure (in [2] the effect of the moisture content was ignored). All these questions can be solved only based on experimental information.

COMPLEX EXPERIMENT ON THE DETERMINATION OF THE REFRACTION OF STARS AND THE METEOROLOGICAL PARAMETERS OF THE ATMOSPHERE

In this work we use the results of simultaneous astronomical and aerological observations, carried out in October of 1968 and 1972. The region where the measurements were performed was a slightly hilly semidesert with an argillaceous-rocky underlying surface.

The astronomical observations were carried out under conditions of "open air" with the help of a UV 2"/2" high-precision universal instrument and the recording apparatus of an autonomous time service. The moments of observation were recorded on the strip chart of a plotting chronograph with an accuracy of 0.02 sec. Referencing to the reference time scale was carried out using an impulsive attachment for automatic reception of signals of the exact time by an all-wave receiver. The point of observation was situated 360 m above sea level.

To determine the astronomical refraction from observations of rising and setting bright stars, planets, and the sun the difference between the zenith distances of the stars measured at a definite time and the computed (true) values was determined. The zenith distances of the stars at a fixed time were measured in arbitrary hour angles by an absolute method, and in so doing the horizontal and vertical circles of the instrument were oriented based on the observations of the North Star. The "location of the zenith" on the vertical circle was determined before and after the series of observations. In the method used the zenith distances of stars were measured in a wide sector of angles (±45°) from the first vertical of the eastern and western parts of the celestial sphere. The true zenith distances of the stars were calculated from the known times and ephemerise values of the coordinates. The error in these calculations is attributable to the errors in the determination of the moment of observation, errors in the latitude and longitude of the measurement point, and errors in the coordinates of the star.

In measurements of astronomical refraction near the horizon in the interval of elevation angles 0-5° the turbulent jittering of the images and the dispersion of the refractive index of the atmosphere made the main contribution to the error in the measurement of the visible zenith distance. Part of the total error is also attributable to the instrumental measurement errors (Table 1).

The observations were accompanied by measurements of the temperature, pressure, and moisture content of the air at the objective of the instrument . The thermodynamic variables of the atmosphere (temperature, pressure, and relative humidity) were determined by the method of aerological (pilot-balloon) sounding up to an altitude of about 30 km with the help of the "Meteorite— RKZ-2" system, which enabled measurements with high altitude discretization (every 100 m up to 2 km; 101 points up to 25 km). The radiosondes were launched at the time of the astronomical observations near the point of observation.

RESULTS OF THE RECONSTRUCTION OF THE ATMOSPHERIC PARAMETERS FROM EXPERIMENTAL DATA

In reconstructing the parameters of the atmosphere from measurements of the refraction it is proposed that the reconstructed profiles be compared with the aerological sounding data, which is a generally accepted method for checking the effectiveness of remote sensing methods, in spite of the fact that the probes themselves determine the atmospheric variables with a certain error.

For this reason, in order to make use of the reconstruction algorithms developed, the measured values of the refraction must first be compared with the values calculated from the corresponding aerological data in the spherically symmetric approximation. Aside from the errors in measurements of the refraction and errors in the aerological data, the disagreement between the computed and measured values of the refraction is also caused by errors in the calculation of the refraction and also the effect of horizontal nonuniformities both directly on the refraction and on the indications of the probe carried by the wind. The inverse problem can be solved and the results of reconstruction compared with the data from aerological measurements only if the difference between the measured and computed values of the refraction satisfies certain requirements, determined in [2], imposed on the measurement accuracy.

Thus, closeness of the values of the computed and measured refraction presumes quite accurate measurements, an accurate computational algorithm, and correspondence of the atmosphere to the spherically symmetric approximation. All these requirements are, at the same time, prerequisites for successful solution of the problem of reconstructing the parameters of the atmosphere from measurements of the refraction.

The accuracy of the calculation of refraction from the viewpoint of the solution of the inverse problem determines the accuracy with which the kernel of the integral equation is determined. To calculate refraction based on the formula (1) the values of the refractive index calculated at 101 altitudes from 0 to 25 km from probe data on the temperature and moisture content of the atmosphere were used. In order to match the meteorological parameters the pressure profile was calculated based on the barometric formula. We present the corresponding expressions:

$$N = k_1(\lambda) P/T + k_2 E/T, \tag{3}$$

$$\rho(h) = \frac{M}{R_0 (1 + 0.378 E/P)} \frac{P}{T}, \tag{4}$$

$$P(h) = \int_h^\infty g(h') \rho(h')\, dh', \tag{5}$$

where ρ is the air density, E is the partial pressure of the water vapor, g *is* the acceleration of gravity, k *is a* constant which depends weakly on the wavelength λ; k~ is a constant (the values of k_1 and k_2 are given in [7]), M *is* the molecular weight of air, and R_0 is the universal gas constant. The dependence of the acceleration of gravity $g(h)$ on the latitude of the point of observation was taken into account in the calculations. The effective wavelength of the measurements in (3), taking into account the reddening at small elevation angles, was chosen as $\lambda = 0.6$ μm. The integration above 25 km was carried out using the standard model of the atmosphere (the variations of the contribution to the refraction of this layer equal less than 0.1")- The calculations showed that the altitude discretization is entirely sufficient for accurate integration.

Table *2* shows the results of the comparison of the measured values of refraction for the star Arcturus and the corresponding values of the refraction calculated from aerological sounding data.

It is evident that the measured refraction coincides with the computed value with an accuracy sufficient for successful reconstruction of the atmospheric variables from the values of ε_{meas} The results of the reconstruction of the index of refraction, the pressure, and the temperature presented in Table 3 for selected altitudes show that the profiles of the atmospheric variables determined from the data in Table 2 are in good agreement with the corresponding aerological sounding data. In particular, the characteristic feature of the profile—the temperature inversion at the ground—is reconstructed.

In the reconstruction the profile $P(h)$ was calculated from the profile $N(h)$ based on the relations (3) -(5), and in addition the model moisture-content profile, decreasing exponentially from the value E_0 at the ground with the characteristic scale $h_0 = 3$ km,

Table 1

Estimate of the Contribution of the Basic Sources of Error in the Determination of the Refraction and Estimates of the Maximum Total Error 5e

ϑ. deg	Instrumental, angular seconds	Turbulent jittering, angular seconds	Variance, angular seconds	Calculation of the zenith distance, angular seconds	δε, "
5	2	1,7	1,4	0,6	3
2	2	2,4	9	0,7	4
1	2	3,3	2,6	1	5
0,5	2	4	3	1,1	6

Table 2

Comparison of the Measured and Computed Values of the Refraction

ϑ, "	ε_{meas}	ε_{comp}
5426	1238	1241
7224	1077	1078
9023	944	949
10829	849	846
12623	760	761

Table 3 Results of Reconstruction of the Atmospheric Variables

H. km	N	N_1	N_2	T, K	T_1	T_2	P, mbar	P_1,	P_2
0	276,9		–	280,0			985,6	–	–
0.2	263,1	261.2	2b2.7	286.5	288.6	287.0	B57.8	958.2	958.3
0.5	255,8	254.0	256.0	284.3	285.7	283.5	924.1	925.3	925.6
1	243,7	243.7	242.8	281 .0	281.6	279	870.n	872.0	870.9
2	220,5	221.3	[illegible]	274.7	274.5	272.9	769.5	771.8	770.9
3	198,8	199.3	?97'.3	268.8	268.7	266.8	678.7	680.5	678.8
4	179,3	179.3	180.1	262.0	262.6	260.5	596.8	598.2	596.0
5	100,8	161.?.	162.3	256.2	256.2	253.3	523.2	524.7	522.4
6	145,2	145.0	143.7	247.8	248.8	246.1	457.0	458.2	455.2
7	131 ,0	130.2	130.9	238.8	'40 9	237.7	397.3	398.5	395.2
8	117,0	116.8	117.1	231.2	232.8	229.5	343.6	345.2	341.5
9	103,8	104,7	104,3	224,5	224,6	221,1	295,9	297,6	293,8

Note: N, I, and P are the probe values, N_{1}_, T_1, and P_1 are the values reconstructed by the method of statistical regularization, and N_2, T_2, and P_2 are the values reconstructed by the method of solution on a compact set of monotonic functions.

$$E(h)=E_0 \exp(-h/h_0). \qquad (6)$$

$$T(h)=(k_1 P+k_2 E)/N. \qquad (7)$$

was substituted into (3) and (4). The temperature profile was reconstructed by solving (3) with the known profiles *N(h)* and *P(h)* based on *T(h)* given by

We note that in the reconstruction the covariational matrix of the index of refraction B_{NN}, obtained based on an ensemble corresponding not to the conditions of observation, but rather to

Table A

Measured and Computed Values of the Refraction
for the Example in Fig. 3

ϑ_0''	ε_{meas}''	ε_{comp}''	ϑ_0''	ε_{meas}''	ε_{comp}''
2114	1676	1663	9266	933	936
3352	1466	1477	10492	867	865
4581	1308	1326	11 671	803	806
5777	1197	1203	12907	749	751
6937	1094	1101	14305	694	696
8091	1006	1013			

Table 5

Measured and Computed Values of Refraction for
Observations of the Star Vega

ϑ_0''	ε_{meas}''	ε_{comp}
3597	1444	1443
5399	1242	1241
8990	952	954
10793	852	851
12597	762	765

Table 6

Measured and Computed Values of Refraction for
Observations of the Sun

ϑ_0''	ε_{meas}''	ε_{comp}''
4366	1429	1423
6842	1152	1150
9123	981	973
11835	814	819

the winter conditions in the central part of the European territory of the USSR, was used but this did not significantly affect the quality of the reconstruction. This question, important from the practical standpoint, about the possibility of employing in the reconstruction algorithms covariational matrices obtained for a different meteorological ensemble was specifically investigated. The results obtained show that these matrices are sufficiently universal in the sense studied. This is illustrated by another example of the reconstruction of the temperature profile from measurements of the refraction of the star Arcturus, presented in Fig. 2. The reconstruction was carried out by the method of statistical regularization using the matrices 5 for summer and winter ensembles of the central part of the European Territory of the USSR. Table 4 shows the corresponding values of the measured and computed refraction.

It is evident that both matrices give close results and are in good agreement with the profile determined with the probes.

Generalization of the experimental data obtained shows that the errors noted and other factors giving rise to a disagreement between the measured and computed values of the refraction in different cases can make a substantially different contribution. In particular, situations are encountered in which at all angles the computed refraction is very close to the measured refraction, which confirms the good accuracy of the calculations and also the high quality of the experiment. This is illustrated by the example presented in Table 5 and Fig. 3 (observations of the refraction of the star Vega).

The sensitivity of the method to the reconstruction of the features in the distribution of the parameters in the boundary layer of the atmosphere, such as the temperature inversion, is of great interest. Figure 4 shows an example of the reconstruction of a deep temperature inversion from measurements of the refraction of the sun (method of statistical regularization). The corresponding measured and computed values of the refraction are presented in Table 6.

CONCLUSIONS

The results of the analysis of the experimental data, in particular comparison of the refraction computed from aerological data and the measured refraction, as well as comparison of the profiles of the atmospheric variables reconstructed by two different methods with profiles determined by probes, show that the accuracy of the measurements of the refraction, the accuracy of the calculation of the kernel of the integral

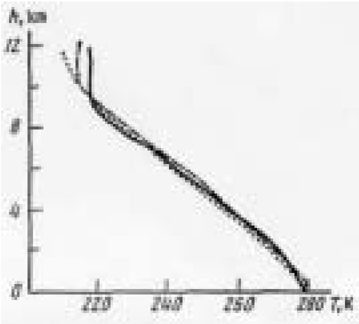

Fig. 2. Comparison of the temperature profile obtained with a probe (solid curve) and profiles reconstructed from measurements of the refraction of the star Arcturus by the method of statistical regularization: the broken curve was obtained using the summer matrix £„ and the dots were obtained using the winter matrix.

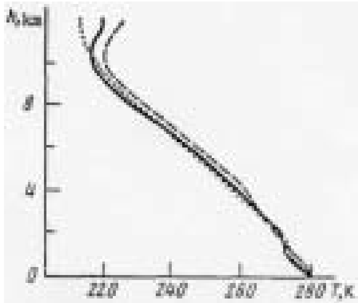

Fig. 4. Reconstruction of the temperature profile with a strong inversion from measurements of the refraction of the sun by the method of statistical regularization (broken curve). The solid curve is the profile determined by the aerological sounding.

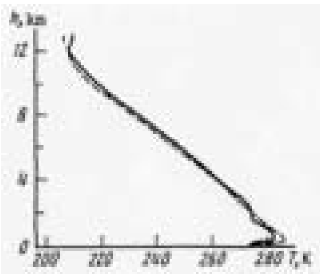

Fig. 3. Comparison of the temperature profile obtained with aerological sounding (solid curve) and the profiles reconstructed from measurements of the refraction of the star Vega (Table 5). The broken curve was obtained by the method of statistical regularization, and the dots were obtained by reconstruction based on a set of monotonic functions.

equation and the degree of correspondence between the real atmosphere and the spherically symmetric approximation under the conditions of the experiment satisfy the requirements formulated in [2] for successful reconstruction of the atmospheric parameters. The accuracy of the reconstruction for the accuracies realized in the refraction measurements corresponds to the theoretical estimates of the accuracy obtained in [2] and is comparable to the best theoretical estimates of the reconstruction accuracies achieved in the method of ground-based microwave radiometry. Temperature inversions are reconstructed well. This indicates the effectiveness of the algorithms developed for reconstructing the variables of the atmosphere (especially based on the method of statistical regularization) and shows that refractometric measurements are useful in problems of long-range sounding of the atmosphere.

The method must be further studied under different conditions, and statistical samples must be accumulated. In a number of situations (irregular underlying surface, atmospheric fronts, etc.) it may be assumed that the horizontal nonuniformities will substantially affect the refraction. Then the nonuniformities themselves could be the object of investigation by remote refractometric methods.

It is of great interest to study the inverse problem of astronomical refraction in the radio range, because of the all-weather applicability of radio methods, the possibility of using the satellite transmitters, and also the possibility of studying the stratification of the moisture content.

It has not been excluded that the initial reconstruction algorithms developed here can be improved, just like the measurement method itself can be improved. Work in this direction can substantially elucidate the interrelationship of refraction and stratification of the atmosphere.

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