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On a Reconstruction of Meteorological Parameters from Intra-Atmospheric Measurements of Optical Refraction of Cosmic Sources

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The physical and mathematical aspects of the statement and solution of an inverse refractometric problem—the reconstruction of the refractive index and meteorological parameter profiles from intra-atmospheric measurements of optical refraction of extraterrestrial sources—are investigated. It is shown that if the observer is within the atmosphere, then the inverse problem is well defined for negative elevation angles and is mathematically incorrect for positive angles. The possibility of reconstructing the vertical temperature and pressure profiles is investigated by means of a numerical modeling of the refractometric experiment on a computer, and estimates are given for the errors of the solution of the corresponding inverse problem. It is shown that in a spherically stratified atmosphere severe distortions of the solar image are observed at sunrise (and sunset) for negative elevation angles and these distortions reflect the structure of the temperature field below the observation point.

When the sun is viewed low on the horizon, we can usually see distortions of the disk, caused by refraction. Thus, a large number of sunset photographs were obtained using a 6-m focal length telescope in the Castel Gandolfo Observatory [1], located at an altitude of 435 m above sea level 20 km from the coast of the Mediterranean Sea (the distance to the horizon during the sunset observations was about 100 km_m).

If the rarely encountered waveguide propagation conditions are ignored [2, 3], then refraction theory in the approximation of a spherically symmetrical atmosphere can apparently be used to describe the refraction effects that are observed at small elevation angles of heavenly bodies. In this approximation it is assumed that the refractive index n in the region where the effect of the atmosphere on the curvature of light rays is significant depends only on the distance r from the center of the planet: $n = n(r)$.

For a spherically symmetrical atmosphere we can state the inverse problem—determine the $n(r)$ dependence from measurements of the refraction angles ε. This is of interest for meteorology, atmospheric physics, and a rapid determination of the propagation characteristics of electromagnetic waves in various bands since the index of refraction $N = (n - 1) \times 10^6$ in the optical and infrared spectral regions is proportional to the air density ρ

$$N = A_\lambda \rho \tag{1}$$

where A_λ is a coefficient that is slightly wavelength dependent [4] (for $\lambda = 0.6$ μm $A_\lambda = 2.25 \times 10^5$ $g^{-1} \cdot cm^3$). Thus, refraction measurements open up new possibilities for remote sounding of the atmosphere; this was demonstrated by the results of refraction measurement experiments on the "Salyut-6" orbiting spacecraft [5, 6].

The direct and inverse problem of the observation conditions for small-scale perturbations of the vertical air density, temperature and pressure profiles from the results of refraction measurements from the earth's surface and various altitudes, including observations from low altitudes of ~10-500 m, are formulated and solved in this paper. A numerical modeling of the refractometric experiment using sunrise observations at optical wavelengths is employed as the investigation method in this paper. The question of the degree of conditionality of the inverse refraction problem is also investigated in this paper for different observation conditions (above and below the level of the observer, located within the atmosphere).

1. BASIC RELATIONS FOR ATMOSPHERIC REFRACTION

The ray path geometry for the problem being considered is shown in Fig. 1. If the refractive index field has spherical symmetry* $n = n(r)$, then

*See [8] for the applicability of this condition.

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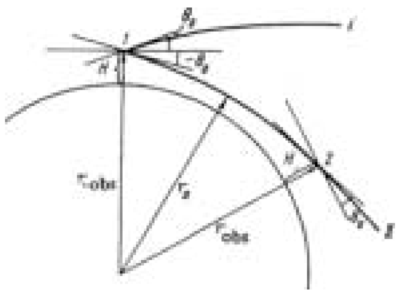

Fig. 1. Ray geometry of refraction experiment from altitudes within the atmosphere.

Snell's law for each ray can be written as [7]:

$$rn(r)\sin\varphi = p = \text{const}, \tag{2}$$

where $\varphi = \varphi(r)$ is the zenith angle of the ray.

A. Let us first consider refraction observations at positive elevation angles θ_0 at the observation point r_{obs}, $\theta_0=\pi/2-\varphi(r_{obs})$. Ray I in Fig. 1 corresponds to this case. The observed refraction angle ε_+ for each θ_0 can be calculated from the formula

$$\varepsilon_+ = -p\int_{r_{obs}}^{\infty}\frac{d\ln n}{dr}\frac{dr}{[(nr)^2-p^2]^{1/2}}, \tag{3}$$

$$p = r_{obs}\,n(r_{obs})\cos\theta_+.$$

Equation (3) relates the measured refraction angle $\varepsilon(p)$ to the unknown vertical profile $n(r)$. If the value of $n(r_{obs}) = n_{obs}$ *is* measured at the observation point at the same time as the refraction value and r_{obs} is assumed to be known, then solving Eq. (3) will give the desired $n(r)$ profile.

Integrating Eq. (3) by parts, after replacement of the variable of integration $rn(r) = x$ we o

$$\frac{\varepsilon_+(p)}{p} - \frac{\ln n_{obs}}{(r_{obs}^2 n_{obs}^2-p^2)^{1/2}} = -\int_{r_{obs}n_{obs}}^{\infty}\frac{x\ln n(x)}{(x^2-p^2)^{3/2}}\,dx, \tag{4}$$

which is the integral Fredholm equation of the first kind for the unknown function $\ln n(x)\approx 10^{-6}N(x)$.

The solving of this equation is a typical incorrect (in the classical sense) problem [9]. In order to solve Eq. (4) it is necessary to resort to some method of regularization. The absence of an appreciable effect of temperature stratification on the solar image in this case (see Sect. 2 of this paper) emphasizes the difficulties of solving Eq. (4).

Let us note that the problem of determining the vertical profiles of the refractive index in planetary atmospheres with bistatic radar of the planets (for positive sounding angles), which is similar in ray geometry and initial equations although not completely the same as the problem being considered in this paper, was discussed in [10]. Therefore, all of the comments made above concerning the character of the inverse refraction problem for positive sounding angles also apply to the problem statement in [10] in full measure.

It is important to note, however, the fact that just a slight raising of the observation point (even by a few meters) can make it possible to perform refraction observations at negative elevation angles, at which a completely different situation is possible.

B. The geometry of intra-atmospheric refraction observations at negative elevation angles ($\theta_0<0$) is shown in Fig. 1 (ray II).

For angles $\theta_0<0$ the refraction angle ε_- can be calculated from the formula

$$\varepsilon_- = -2p\int_{r_0}^{r_{obs}}\frac{d\ln n(r)}{dr}\frac{dr}{[(nr)^2-p^2]^{1/2}} - p\int_{r_{obs}}^{\infty}\frac{d\ln n(r)}{dr}\frac{dr}{[(nr)^2-p^2]^{1/2}}, \tag{5}$$

where the perigee distance of the ray is $r_0 < r_{obs}$, $p = r_0n_0 = r_{obs}n_{obs}\cos\theta_0$, and $n_0 = n(r_0)$. The first term in Eq. (5) describes the refraction of a ray as it propagates between points 1 and 2, where $r \le r_{obs}$; the second term describes the rest of the path, where $r > r_{obs}$. The second term in Eq. (5) is identical to Eq. (3); this is a consequence of the symmetry of the rays with respect to the turning point $r = r_0$. It follows from this that the second term in Eq. (5) can be obtained from refraction measurements at $\theta_0 > 0$, and therefore the equation

$$\frac{\varepsilon_-(p)-\varepsilon_+(p)}{2p} = -\int_{p}^{r_{obs}n_{obs}}\frac{d\ln n(x)}{dx}\frac{dx}{(x^2-p^2)^{1/2}}, \quad x=rn(r) \tag{6}$$

must be solved in order to find $n(r)$ in the interval $r_0 < r < r_{obs}$. Equation (6), unlike Eq. (4), is a Volterra integral equation and it can be reduced to an Abel equation by elementary transformations [11]. The exact solution of Eq. (6) is expressed by the following formula

$$\ln\left[\frac{n(x)}{n_{obs}}\right] = \frac{1}{\pi}\int_{x}^{r_{obs}n_{obs}}\frac{\varepsilon_-(p)-\varepsilon_+(p)}{(p^2-x^2)^{1/2}}\,dp, \quad r=\frac{x}{n(x)}. \tag{7}$$

By having the exact solution, we can easily prove that small errors in the c measurements lead to small errors in the n values obtained from Eq. (7). With a decrease in the correlation

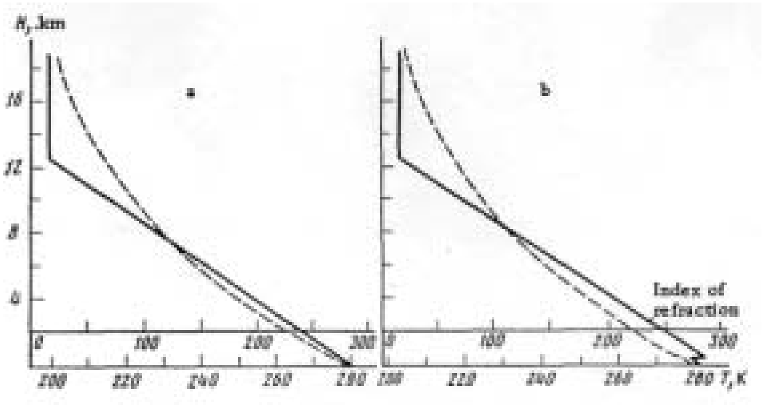

Fig. 2. Examples of the altitude dependence of the temperature *T* (solid curves) and the index of refraction $N = (n - 1) \times 10^6$ in the optical band (dashed curves): a) *T* and *N* profiles close to the standard profiles; b) *T* and *N* profiles in the presence of a small temperature inversion near the earth's surface.

radius of the measured errors $\Delta\varepsilon$ the error of the reconstructed *n* values also decreases. All of this leads to an overall good conditionality of the problem of reconstructing the refractive index from measurements of the refraction of cosmic sources at negative elevation angles.

Let us note that at observation altitudes of ~100 m, when opposite edges *of* the sun are observed simultaneously for both positive as well as negative elevation angles and for all observation points when $\theta_0<0$ corresponding image points exist with values $\theta_0>0$, that are equal in absolute magnitude, it is possible to determine the difference $e_- - e^+$ for Eq. (6) from one photograph by using the formula

$$\varepsilon_-(\theta_0) - \varepsilon_+(\theta_0) = \sqrt{d^2-a^2}+\sqrt{d^2-b^2}-2\theta_0, \tag{8}$$

where *d* is the angular radius of the sun, and *a* and *b* are the abscissas, measured from the center of the disk, of the image points with positive and negative angles θ_0, respectively. In this case there is no need to determine the absolute values of e^+ and e_- separately; this requires a rather precise time correlation of the measurements and influences the overall accuracy of the results. In order to reconstruct the profiles of the meteorological elements down to the surface level in this situation it is necessary that the sun be tangent to the horizon.

QUANTITATIVE RESULTS OF SOLVING THE

Numerical experiments on a computer have made it possible to obtain clear confirmation of the general results presented above. Temperature profiles close to the standard profile (Fig. 2a) as well as profiles with different inversions, including small inversions at heights of ~250–400 m (Fig. 2b), were used in modeling the refraction experiment. In the meteorological realizations depicted in Fig. 2, the surface pressure was 1000 mbar. The vertical profiles of the index of refraction *N* at optical wavelengths were calculated for all of the data using Eq. (1). For purposes of clarity the results of the *N* calculations were used to construct the solar image (Fig. 3) at sunrise for observers located at heights *H* = 500 m — above the inversion (observer *A)*—and *H* = 25 in — below the inversion (observer *B).* The solar images were constructed on a plotter attached to the computer (BESM-6).

A comparison of the images of Fig. 3 shows that the pattern, which observer *A* should see, differs drastically from that which observer *B* can see. Observer *A* can easily discern the difference between a smooth profile and a profile with the presence of an inversion, whereas the patterns are nearly indistinguishable for observer *B.* This result graphically demonstrates the difference in the sensitivity of refractometric measurements to variations of the refractive index for observations at positive and negative elevation angles. The data obtained indicate that for observations at positive elevation angles no features (except for the overall compression) are detected in the image of the disk,* and the entire variety of distortions in the solar image near the horizon are observed only at negative elevation angles.

The possibilities of reconstructing vertical temperature profiles from refractometric data were investigated on temperature profiles having a large number of fine details below the observation point. One such profile is shown in Fig. 4. The numerical experiment consisted of solving the direct (a computation of e from known meteorological

*This conclusion is also valid for very large temperature inversions.

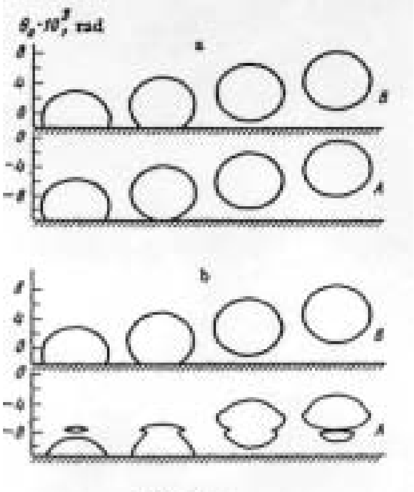

Fig. 3

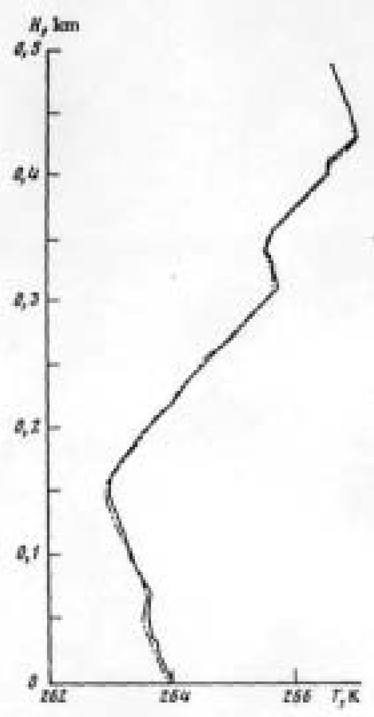

Fig. 4

Fig. 3. Dynamics of the image *of a* rising sun, seen through the earth's atmosphere. Modeling of the ground experiment for the altitude profiles of the temperature shown in Fig. 2 (a and b) . Observer *A is* situated at an altitude *H* = 500 m (above the temperature inversion in case *b),* while observer *B* is at an altitude *H* = 25 m (below the temperature inversion in case *b).*

Fig. 4. Example of a reconstruction of the temperature altitude profile for atmospheric refraction observations with an error $\delta\varepsilon$ = 1% from an altitude *H* = 500 m (numerical experiment). The continuous curve is the "true" value of *T(h),* and the dashed curve is the reconstructed profile.

data) and inverse (reconstruction of the profiles of *N, T* and the pressure *P* from the $\varepsilon(p)$ relationship) problems. An altitude-uncorrelated random error with a normal distribution, the variance of which was equal to the square of the rms measurement error being modeled was "sketched" by means of a random number generator in the computer onto the "true" refraction value e calculated by solving the direct problem for each meteorological realization used.

An example of a reconstruction of the temperature profile for an observation height of 500 m is shown by the dashed curve in Fig. 4. Simulated data with an altitude interval equal to ~10 m was used for the reconstruction; this corresponds to an angular increment of ~20". The errors of the modeled measurements were assumed to be statistically independent with an rms value of ~1%, this corresponds to an accuracy of about 15" for the measurements of the optical refraction angles. The *N* and *T* profiles were reconstructed both by means of relationship (7) and also by solving the inverse problem numerically using the method described in [12]. In the latter case Eq. (6) was solved initially for the derivative *dn/dr,* and then the altitude profiles of « (from the known values on $n(r_{obs})$ and *dn/dr*)*, P* (from the barometric formula having the form $dP/dr = \text{const}(n - 1)$ when Eq. (1) is taken into consideration) and *T* (from relationship (1) using the known values of *n*(r) and *P(r)*) were determined. The temperature profiles, reconstructed by the described method, duplicate the original profiles within an error of less than 0.1 K, and the corresponding errors in the determination of the atmospheric pressure were no greater than 0.1 mbar (taking into consideration that the barometric formula was used for the pressure at the average humidity values at the corresponding altitudes). The error of the refractometric determination of the atmospheric parameters can increase several fold because of the effect of turbulent fluctuations. If humidity is ignored, then the refraction measurements make it possible to determine the virtual temperature.

The use of refractometric methods for reconstructing meteorological elements in the lower atmospheric layers is encouraged to a considerable extent by the possibilities of very precise knowledge of the kernel of the integral Eq. (6), which is determined by the refracting properties of the atmosphere. Thus, the uncertainty in the knowledge of the index of refraction due to uncertainty in the coefficient A_λ. (see, for example, the data summary in [13]) does not exceed ~$7\cdot 10^{-2}$ unit of *N* in the lower atmospheric layers; this corresponds to a relative error of δN~0,03%.

Moreover, it must be stressed once again that the discussion presented above was based on the assumption of spherical symmetry for the refractive index field. The physical meaning of this condition consists of the fact that an atmospheric layer of thickness *h*. must have a horizontal extent of the order of $\sim 2(a_{eq}\, h)^{1/2}$ along a ray, where a_{eq} is the equivalent radius of the earth. This condition is quite stringent, especially if the unavoidable influence of turbulence *is* taken into account. An investigation of the influence of nonsphericity and of turbulence on the reconstruction of the average (over the region in which the image of cosmic sources is formed) profiles is a rather complicated problem which is still awaiting solution.

As the estimates and the results of the performed numerical experiments show, refractometric measurements can make the extremely small vertical gradients of air density in the surface layer of the atmosphere accessible to remote measurement. This is all the more important because of the fact that direct measurements in the atmospheric layer above water surfaces are quite complicated because of the smallness of the vertical gradients, which can lead to large errors. This explains to some extent the dearth of information on the results of such measurements [14].

In principle, refraction measurements provide an average (with a certain weighting factor) value of the refractive index along the ray. The spatial averaging can prove to be a disadvantage of the method in some cases. However, as a rule, the average values are of interest and in this case, if there are bases for assuming horizontal homogeneity, a spatial averaging can lead to results that are statistically more stable than local measurements involving an unavoidable time averaging.

It must be stressed that the above-presented proof of sufficient conditionality of the reconstruction problem of the vertical refractive index profiles is based on the results of a numerical experiment and is done at a physical level of rigor rather than mathematical.

On the basis of the results obtained let us also point out the possible prospects for using refractometric methods at microwave frequencies for a remote determination of atmospheric meteorological parameters under various weather conditions including cloudy situations.

CONCLUSION

Let us briefly summarize the principal conclusions of this work.

1. The inverse refraction problem in a spherically symmetrical atmosphere is well defined for observations of extraterrestrial sources at zenith angles greater than 90° (for negative elevation angles) and is incorrect for zenith angles less than 90° (at positive elevation angles).

2. On the basis of a numerical experiment it has been shown that the vertical profiles of the virtual temperature and pressure below the observation point can be reconstructed with a small error (of the order of and less than 0.1 K and 0.1 mbar, respectively) with very moderate requirements on the accuracy of the measurement of the optical refraction angles (~15").

3. For the conditions of a spherically stratified atmosphere the most complicated deformations of the solar image at sunrise or sunset are observed at negative elevation angles and are the consequence of the complicated structure of the temperature field in the perigee region of the ray at altitudes less than the altitude of the observation point. For observations at positive elevation angles the deformations of the solar image reflect the structure of the temperature field above the observer level on an average basis only (along the ray) and are evident only in the form of a greater or less flattening of the disk.

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