Izvestiya, Atmospheric and Oceanic Physics

Vol. 20, No. 8, 1984
UDC 551.501.7:525.723

Possibility of Determining the Meteorological Parameters of the Atmosphere from Radio and Radio-Optical Measurements of the Refraction of Cosmic Sources

K. P. GAYKOVICH Scientific-Research Institute of

Radio Physics, Gorky

The use of combined measurements of refraction in the optical and radio regions as well as refraction in the radio region only in order to reconstruct different meteorological parameters, including the moisture content, is studied. The effect of measurement errors on the -accuracy of reconstruction of the moisture content, pressure, temperature, and index of refraction is studied with the help of a numerical experiment which made it possible to determine the required experimental conditions. Variants of both intra-atmospheric and cosmic experiments are examined.

At the present time a great deal of attention is being devoted to the possibilities of making remote measurements of the meteorological parameters of the atmosphere with the help of measurements of the refraction of radiation from cosmic sources in the earth's atmosphere. Different aspects of the formulation and solution of inverse refractometric problems for both cosmic and intra-atmospheric observations in the optical range were examined in [1-4]. It was established that for a spherically-layered atmosphere the problem of reconstructing the altitude profile of the index of refraction is properly posed for altitudes below the altitude of the observer, where the index of refraction is related to the refraction by the inverse Abel transformation. The problem of determining the index of refraction above the observer's altitude is not properly posed, since the quantities being measured and reconstructed are related by an integral equation that reduces to a Fredholm equation of the first kind [3].

In the references indicated above, refraction in the optical range, where the index of refraction is proportional to the air density, was examined. The pressure and temperature profiles could be reconstructed with the help of the barometric formula by observing the setting of the sun and moon through the earth's atmosphere.

In the radio region the index of refraction also depends on the amount of moisture in the air, and for this reason it was suggested in [2, 3] that radio refraction be used to determine the content of this atmospheric component. Measurements of refraction in the radio range differ from measurements in the optical range by a number of peculiarities. Thus, in the radio range (at least in the transmission bands), ratio sources can be observed in the entire range of angles of elevation and even in the presence of clouds, whereas in the optical range, even in clear weather, for very low angles of elevation only the sun can be observed in practice. Radiofrequency measurements, however, have their own difficulties: the receiving apparatus is more complicated, it is more difficult to achieve the required angular resolution, and the effects of diffraction by inhomogeneities in the atmosphere are greater. Nevertheless, such measurements can greatly supplement and refine the data obtained in the optical region.

INTRA-ATMOSPHERIC COMBINED MEASUREMENTS OF RADIO AND OPTICAL REFRACTION

In analogy to [3], where the optical experiment was examined, we select the observer altitude *H* = 500 m. For the analysis we shall use model profiles of the temperature and moisture content with a very complicated structure. The geometry of the rays for this case are shown in Fig. la. The quantities being measured here are the optical and radio refraction e as a function of the angle of elevation θ_0. As shown in [3], the relationship between the refraction and the index of refraction is expressed, up to a substitution of variables, by the following relations:

$$\varepsilon_+(\theta_0) = -\int_{r_H}^{\infty} \frac{d\ln(n)}{dr} \frac{n_H r_H \cos\theta_0}{\sqrt{(nr)^2 - (n_H r_H \cos\theta_0)^2}} \quad (\text{Ray I}),$$

$$\varepsilon_-(\theta_0) = -\int_{r_H}^{\infty} \frac{d(\ln n)}{dr} \frac{n_H r_H \cos\theta_0}{\sqrt{(nr)^2 - (n_H r_H \cos\theta_0)^2}} - \qquad (1)$$

0001-4338/85/2008-0001$18.00/1

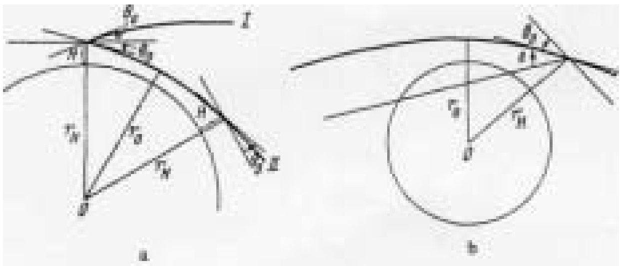


Fig. 1.

$$-2\int_{r_0}^{r_H}\frac{d\ln(n)}{dr}\frac{n_0 r_0\,dr}{\sqrt{(n r)^2-(n_0 r_0)^2}} \quad \text{(Ray II)}$$

$$n_0 r_0 = n_H r_H \cos\theta_0,$$

where $n(r)$ is the index of refraction and $n_0=n(r_0)$, $n_H = n(r_H)$. The inversion formula in this case has the form

$$n(\theta(r)) = n_H \exp\left[\frac{1}{\pi}\int_0^{\theta}\frac{\varepsilon_-(\theta_0)-\varepsilon_+(\theta)}{\sqrt{\cos^2\theta_0-\cos^2\theta}}\sin\theta_0\,d\theta_0\right], \qquad (2)$$

while the transformation form $n(\theta)$ to $n(r)$ is determined by the relation $\cos\theta = nr/n_H r_H$. The solution of the inverse problem (2) permits obtaining the altitude profile of refractive indices both in the optical and in the radio regions for altitudes ranging from 0 to H from measurements of the refraction of a cosmic source (for example, the sun). As is well known, the reduced refractive indices $N=10^{-6}(n-1)$ for the optical and radio regions, with sufficiently high accuracy, have the form [5]

$$N_{opt} = k_1\frac{P}{T} + k_2\frac{E}{T}, \qquad (3a)$$

$$N_R = k_1\frac{P}{T} + k_2\frac{E}{T} + k_3\frac{E}{T^2}, \qquad (3b)$$

where k_1, k_2, k_3 are constants, P is the pressure, E is the partial pressure of water vapor, and T is the absolute temperature. Taking into account the effect of the moisture content, the change in pressure as a function of altitude is described by the equation

$$\frac{dP}{dh} = -\frac{g}{R_a\left(1+0.378\frac{E}{P}\right)}\frac{P}{T}, \qquad (4)$$

where g is the acceleration of gravity and R_a is the gas constant of dry air. Since the contribution of the moisture content in (4) is very small and the second term in (3a) is also small, the pressure can be determined from the expression

$$\frac{dP}{dh} = -\text{const}\,N_{opt}(h). \qquad (5)$$

The altitude distribution of the temperature is then obtained from the obvious equation:

$$T(h) = k_1\frac{P(h)}{N_{opt}(h)}. \qquad (6)$$

Using these data, we find the distribution of the moisture content

$$E(h) = \frac{N_R(h)-N_{opt}(h)}{k_3}T^2(h). \qquad (7)$$

The function £(ft) obtained in this manner can be used in an iterative procedure to obtain a more accurate solution by substituting this quantity into the expressions for N and dP/dh.

Using the method described above, we performed numerical experiments for fixed models of the profiles $T(h)$ and $E(h)$ is obtained directly from (4)). The calculations were performed according to a closed scheme, in which the refraction calculated for the model profiles was to reconstruct them from Eqs, (2) – (7). In this case, the reconstructed profiles of E, T, and P coincided with high accuracy with the starting profiles. In order to investigate the influence of the measurement errors on the accuracy of the solution, a random error was superposed on the computed values with the help of a random number generator and the corresponding errors of the reconstructed meteorological parameters were calculated. It turned out that the errors in the reconstruction of the index of refraction are proportional to the relative error in the refraction measurements, which was modeled by superposing an uncorrelated, normally distributed, random error with zero mean and standard deviation proportional to the quantity ε

$$\delta N_{opt} = 1.2\,\delta\varepsilon_{opt}/\varepsilon_{opt}, \quad \delta N_R = 2.5\,\delta\varepsilon_R/\varepsilon_R.$$

Figure 2 shows the model profiles of E and T, corresponding to values of the quantity $e_-(\theta) - e_+(\theta)$

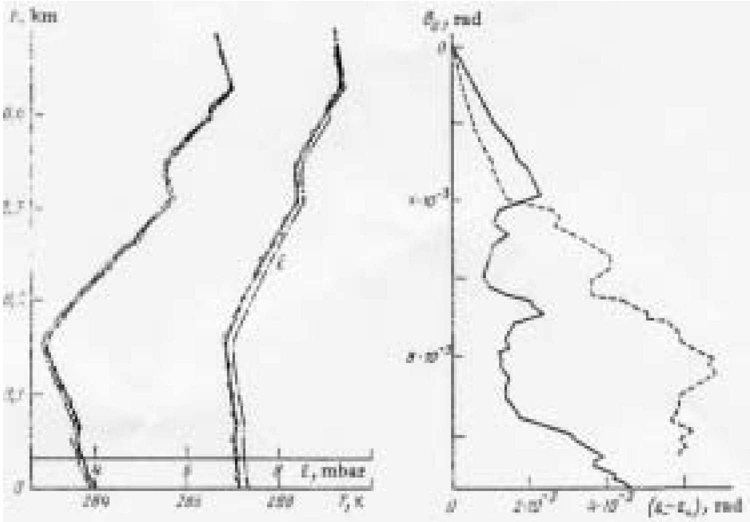
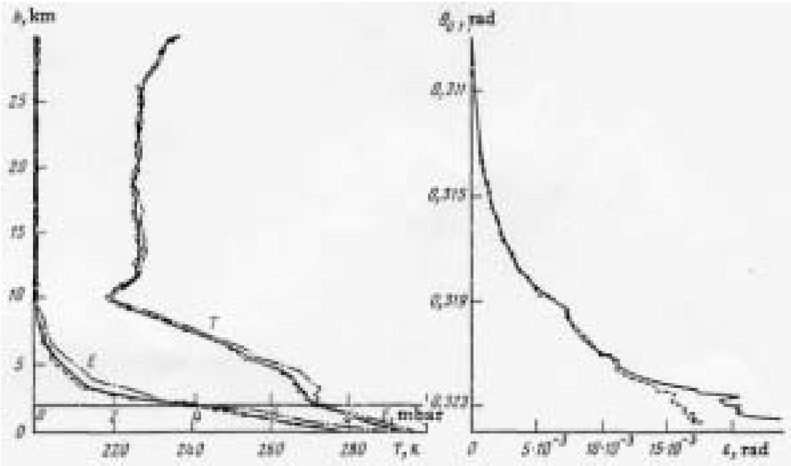


Fig. 2. Intra-atmospheric experiment. Left: starting profiles of the temperature and moisture content (continuous curves); reconstructed profiles with a random error in the measurements of $\delta\varepsilon_{opt}$ = 1%, $\delta\varepsilon_R$ = 10% (dashed curves); reconstructed profiles with systematic error $\delta\varepsilon_{opt} = 3\cdot10^{-5}$ rad, $\delta\varepsilon_R = 3\cdot10^{-4}$ rad (dot-dash curves). Right: measured values of (ε_--ε_+) corresponding to the starting profiles of the meteorological parameters (continuous curve for the radio range and the dashed curve for the optical range).

for optical and radio refraction, as well as the reconstructed profiles of the temperature and moisture content with the modelled measurement errors $\delta\varepsilon_{opt}$ = 1% as a function of the quantity ε_{opt}, itself, while $\delta\varepsilon_R$=10%. For an observer altitude of 500 m the scale of the change in the position of the source according to the angle of elevation is $\Delta\theta_0 = 1,2\cdot10^{-2}$ rad. The discretization with respect to the angle of elevation constituted $\delta\theta_0 \sim 2\cdot10^{-4}$rad, which corresponds to the reconstruction of the profiles of *E* and *T* at 50 altitude points (approximately every 10 m). It is evident from Fig. 2 that the quantity ε-(θ0)-ε+(θ0) ~ $2\div6\cdot10^{-3}$ rad; thus the absolute magnitude of the imposed error in this example constitutes $\delta\varepsilon_{opt} \sim 3\cdot10^{-5}$ rad and $\delta\varepsilon_{Rn} \sim 3\cdot10^{-4}$ rad. We also studied the effect of a systematic error in the measurements on the determination of the meteorological parameters, whose magnitude was set equal to $\delta\varepsilon_{opt} = 3\cdot10^{-5}$ rad and fie $\delta\varepsilon = 3\cdot10^{-4}$. The results of the reconstruction are presented in Fig. 2 and in Table 1 (altitude-averaged errors). From the data presented it is evident that the accuracy of the reconstruction for the random error is higher than with a comparable systematic error, which is explained by the smoothing of the error in (2) and emphasizes the good conditionality of the solution. This is confirmed by a theoretical result [2], which predicts that the error in the determination of the index of refraction decreases as the correlation radius of the measurement errors decreases. We note that the reconstruction error for the systematic error increases downwards from the observer. The estimates obtained show that the high accuracy of the method and comparatively nonstringent requirements for accuracy of refraction measurements (Table 1), in principle, permit using it to study the fine structure of the meteorological parameters. It should be noted that small-scale vertical inhomogeneities can give rise to diffraction smearing of the source in the radio region. Thus, in our case, with λ = 3 mm the vertical resolution is restricted by the scale~15 m (the size of the first Fresnel's zone is $\sim\sqrt{\lambda L}$, where *L* is the path length of the ray). The performance of experiments is complicated by the fact that for the moisture content the conditions of spherical symmetry are most often satisfied over smaller scales than for the pressure and temperature. The method described also makes it possible to study the transverse inhomogeneities of the moisture content and index of refraction in the radio range.

RADIO-OPTICAL MEASUREMENTS OF REFRACTION FROM SPACE

The geometry of this problem differs somewhat from the case examined above and is shown in Fig. 1b.



¥ig. 3. Cosmic experiment. Right: starting profiles of the meteorological parameters (continuous curves), profiles reconstructed from the measurements of optical and radio refraction (dashed curves), profiles reconstructed from measurements of radio refraction only (dot-dash curves). Right: measured refraction (continuous curve for the radio region and dashed curve for the optical region). The simulated error is $\delta\varepsilon_{opt}$=l7%, $_{n}\delta\varepsilon_{R}$=l%.

The relationship between the refraction and the altitude profile of the index of refraction in this case is somewhat simpler, and expressions (1) and (2) reduce to the following relations (analogous, to within a substitution of variables, to those used in [1, 2]):

$$\varepsilon(\theta_0) = -2n_0 r_0 \int_{r_0}^{\infty} \frac{d(\ln n)}{dr} \frac{dr}{\sqrt{(nr)^2 - (n_0 r_0)^2}}, \quad r_H \cos\theta_0 = n_0 r_0, \tag{8}$$

$$n(\theta(r)) = \exp\left[\frac{1}{\pi} \int_{0}^{\theta} \frac{\varepsilon(\theta_0)\sin\theta_0 d\theta_0}{\sqrt{\cos^2\theta_0 - \cos^2\theta}}\right], \quad r_H \cos\theta = rn(r).$$

In order to model the experiment, we used the meteorological sounding data. In analogy to [1, 2, 4], where an optical experiment from space was examined, the height *H* was chosen to be 350 km. Beginning with angles for which the error in the measurements of $\delta\varepsilon$ is comparable to the natural variations in e, in (9) $\varepsilon(\theta_0)$ should be replaced by $\langle\varepsilon(\theta_0)\rangle$, calculated for the mean profile $\langle n(h)\rangle$. The scale of the measurements of the visible angle of the source θ_0 in the atmosphere constitutes $\Delta\theta_0$~ 1Q^{-2} rad. A discretization of $\delta\theta_0$~ $1.5 \cdot 10^{-4}$ rad with respect to the visible angle in the numerical experiment under study corresponds to an altitude discretization of δH~350 mm for the profiles being reconstructed. Figure 3 shows the profiles of the meteorological parameters selected from summertime meteorological sounding data for the central part of the European territory of the USSR and the corresponding values of refraction in the optical and radio regions together with the results of the reconstruction with the random error of the measurements being modeled $\delta\varepsilon_{opt}=\delta\varepsilon_R$=l%. This corresponds (Fig. 3) to a measurement error of $\delta\varepsilon$~10^{-4} rad in the layer h<10 km. The errors of the reconstruction of the meteorological elements constituted: δT = 0.47 K, δP = 0.5 mbar (altitude-average up to 7 km) . The relative errors in the determination of the refractive indices are proportional to the relative measurement errors

$$\delta N_{opt}/N_{opt} = 0.23\delta\varepsilon_{opt}/\varepsilon_{opt}, \quad \delta N_R/N_R = 0.26\delta\varepsilon_R/\varepsilon_R.$$

For $\delta\varepsilon_R$ = 5% , we have δE = 0.25 mbar (altitude-average up to 3 km, where *E* =1.5 mbar). The required technical means and methods for measuring radio refraction, for example, from the Doppler shift of the frequency due to the effect of the atmosphere, now exist [6]. This makes it possible to replace the angular measurements with frequency measurements and to use small antennas. The vertical resolution is limited by the scale of the first Fresnel zone (~70 mm at λ = 3 mm).

The analysis performed above shows that the combined radio-optical measurements of refraction from space permit obtaining reconstructed profiles of the meteorological parameters with high accuracy and altitude resolution. The horizontal averaging along a ray on scales of the order of hundreds of kilometers is a limitation of the method but nevertheless satisfies the requirements of numerical weather forecasting. We note that clouds and atmospheric turbidity sometimes make it impossible to perform optical measurements at angles close to the

Table 1

$\delta\varepsilon_{opt}$	$\delta\varepsilon_R$	δN_{opt}	δN_R	δE,mbar	δT, K	δP, mbar
1%	10%	$2.6\cdot10^{-2}$	$1.2\cdot10^{-1}$	$2.4\cdot10^{-2}$	$2.7\cdot10^{-2}$	$0.9\cdot10^{-3}$
$3\cdot10^{-5}$	$3\cdot10^{-4}$	$6.6\cdot10^{-2}$	$7.7\cdot10^{-1}$	$1.4\cdot10^{-1}$	$7\cdot10^{-2}$	$5\cdot10^{-3}$

Table 2

H, km	Ocean				Continent (summer)			
	<*E*>, mbar	δE mbar	δN_{opt}	δT	<*E*>	δE	δN_{opt}	δT
0	27,2	0,8	2,9	2,5	14,1	1,3	4,7	6,0
2	12,0	0,4	1,5	2,0	7,3	0,7	3,2	5.4
4	5,1	0,2	0,8	1,4	2,6	0,4	1,7	4,0
6	2,1	0,2	0,7	1,7	0,8	0,2	1,3	4.0
8	0.6	0,2	1,0	2,4	0,2	0,2	1,2	4,1

Table 3

Indices	Neglecting moisture content in (10), (11)			Including <E(h)>		
	Ocean	Summer	Winter	Ocean	Summer	Winter
δP<1 mbar	above 10 km	8,5	6	9	8 9	5.5
δT<1 K	11	9,5	7	10		0.5

horizontal and, therefore, to determine the meteorological parameters at the corresponding altitudes. In this connection, we investigated the possibility of reconstructing the meteorological parameters of the atmosphere using only measurements of radio refraction, which is not affected by these factors.

USE OF MEASUREMENTS OF RADIO REFRACTION FOR RECONSTRUCTION OF THE METEOROLOGICAL PARAMETERS OF THE TROPOSPHERE

In the preceding section we showed that by measuring the refraction of radio radiation in the atmosphere from space we can determine from (8) the index of refraction in the radio region $N_R(h)$ with an error of $\delta N_R/N_R \sim 0.3\ \delta\varepsilon_R/\varepsilon_R$, that is proportional to the error in the measurements. As is evident from (3), the altitude distribution $N_R(h)$ depends on all of the meteorological parameters: temperature, pressure, moisture content. It is evident that knowing only $N_R(h)$ it is impossible to obtain the formal solution for determining these characteristics. Nevertheless, they do not make the same contribution to the index of refraction both with respect to altitude and under different meteorological conditions, which opens up a number of possibilities for solving the problem. In particular, the contribution of the moisture content to N_R is large in the lower layers of the atmosphere. Under the assumption that N_R can be determined accurately enough, we shall determine the moisture content from (3) , using the mean-climatic values $\langle N_{opt}\rangle$ and <T>

$$E(h) = \frac{N_R - \langle N_{opt}\rangle}{k_1}\langle T\rangle^2. \tag{9}$$

The mean-square error of the determination of the moisture content $\delta\varepsilon$ will then be determined by the natural climatic variations of the optical index of refraction δN_{opt} and of the temperature σT.

The accuracy of the determination of E was investigated for meteorological ensembles (~100 realizations each), corresponding to different climatic conditions: a) tropical zone of the ocean; b) summer in the central part of the European territory of the USSR; and, c) winter in the central part of the European territory of the USSR.

Table 2 presents the average values of the moisture content $\langle E\rangle$, the mean-square errors in the determination of E from Eq. (9), and the variations in N_{opt} and T. It is evident from the results presented in Table 2 that the relative accuracy of the determination of the moisture content depends on the climatic conditions and on the altitude. Under oceanic conditions the moisture content is determined quite accurately up to an altitude of ~8 km and under summer continental conditions up to an altitude of ~6 km. Under winter continental conditions, the mean value of the moisture content is very low even near the ground ($\langle E\rangle \sim 2.5$ mbar) so that the error in the determination of the moisture content from Eq. (9) becomes comparable to the natural variations. In this case, radio refraction will not provide any information on the moisture content. It should also be noted that in some cases (for example, in tropical cyclones) the deviations of the meterological parameters greatly exceed the mean-climatic variations. Then the error in the determination of the moisture content can increase, but, as is evident from Table 2, even in this case the accuracy of the determination of E will still be quite enough, especially since even the variations of the moisture content under such conditions do not greatly exceed their mean value.

Above some altitudes, the contribution of the moisture content to the index of refraction in the radio region becomes small enough that the profiles of the pressure and temperature can be determined from the radio refraction with good accuracy. This level can be lowered somewhat, if the mean-climatic distribution of the moisture content $\langle E(h)\rangle$ is taken into account. In this case, the pressure is determined from the equation

$$P(h) = \frac{g}{R_a k_1} \int_h^\infty \left(N_R - k_3 \frac{\langle E\rangle}{\langle T^2\rangle} \right) dh'. \qquad (10)$$

Setting $\langle E(h')\rangle = \langle E(h)\rangle \exp(-\alpha(h' - h)$, we can easily estimate the contribution of the second term in the integrand in (10), taking into account the fact that $\langle T^2\rangle$ is virtually independent of altitude. This gives an error in the determination of the pressure in the case when the moisture content is not taken into account. In particular, $\delta P \sim 1$ mbar above the height at which $\langle E\rangle \sim 0.1$ mbar. The temperature profile can be determined from (3) by neglecting the second term

$$T(h) = \frac{k_1 P}{N_R} + k_3 \frac{\langle E\rangle}{\langle T\rangle N_R}, \qquad (11)$$

having first determined $P(h)$ from (10).

Table 3 presents the altitudes beginning with which the error in the determination of the pressure is less than 1 mbar, while the error in the temperature is less than 1 K for the ensembles of meteorological data described above. For altitudes above those presented in Table 3, the pressure and temperature profiles can be reconstructed from radio refraction in the same way as from optical refraction. From Table 3 it is evident that due to the very rapid growth of the moisture content with decreasing altitude (α in the exponent equals ~0.3) the inclusion of the average profile E permits only a small lowering of the boundary of accurate reconstruction of P and T. As is evident from Tables 1 and 2, there exists some range of altitudes in which the contributions of the meteorological parameters to N_R are not separated with the required accuracy.

We note that the presence of the altitude autocorrelation couplings between the meteorological parameters can in principle be used to refine the results, replacing in (9)–(11) the average profiles by the profiles extrapolated from regions where the meteorological parameters are quite accurately determined from radio refraction (E from below, T and N_{opt} from above). However, the errors in the reconstruction in this case decrease insignificantly (by 10–30%).

The estimates presented above are confirmed by numerical experiments using meteorological sounding data obtained under the different climatic conditions indicated above. An example of the reconstruction using information on the average distributions is presented in Fig. 3. Here, the accuracy of the measurements of ε_R was set equal to, as above, $\delta\varepsilon_R = 1$ %, which in the lower layers leads to $\delta N_{Rn} \sim 0.5$. The error in the determination of the moisture content in the layer near the ground is $\delta E = 1.1$ mbar, the altitude averages are $\delta T = 1.7$ K, $\delta P = 4.8$ mbar, and above an altitude of 8 km $\delta P < 1$ mbar, $\delta T < 1$ K.

The problems to be addressed by future research include an investigation of the effect of possible deviations from spherical symmetry in the distribution of the meteorological parameters on the accuracy of the solution of the inverse problem, an investigation of situations with waveguide propagation, as well as an investigation of the possibility of the combined use of radio refraction and other remote methods.

The author is grateful to A. S. Gurvich for discussions of the results obtained and A. P. Naumov for useful remarks.

Received January 20, 1983;
resubmitted May 5, 1983

REFERENCES

1. Gurvich, A. S., V. Kan, L. I. Popov, V. V. Ryumin, S. A. Savchenko, and S. V. Sokolovskiy. "Reconstruction of the atmospheric temperature profile from motion pictures of the sun and moon from the Salyut-6 orbiter." Izv. Akad. Nauk SSSR, FAO.18, No. 1, 3-7 (1982).

2. Grechko, G. M., A. S. Gurvich, A. M. Obukhov, L. I. Popov, V. V. Ryumin, and S. A. Sav-chenko. "Use of refractrometric information in probing the atmosphere from space." Preprint No. 13 (Proceedings of the seminar on the Atmosphere—Ocean—Space under the direction of Academician G. I. Marchuk), VINITI, Moscow (1981).
3. Gaykovich, K. P., A. S. Gurvich, and A. P. Naumov. "Determination of the meteorological parameters from intra-atmospheric measurements 6. of the optical refraction of cosmic sources." Izv. Akad. Nauk SSSR, FAO, 19_, No. 7, 675-687 (1983).
4. Gaykovich, K. P. and A. P. Naumov. "Modeling and statistical analysis of refractrometric method of determining the meteorological parameters from space." Issledovaniye Zemli iz Kosmosa, No. 4, pp. 25-32 (1983).
5. Kolosov, M. A. and A. V. Shabel'nikov. Re-fraktsiya elektromagnitnykh voln atmosferakh Zemli, Venery i Marsa (Refraction of Electromagnetic Waves in the Atmospheres of the Earth, Venus, and Mars). Sov. Radio, Moscow (1976).
6. 0.I. Yakovlev. Rasprostraneniye radiovoln v Solnechnoy sisteme (Propagation of Radio Waves in the Solar System). Sov. Radio, Moscow (1974).