TWO-DIMENSIONAL IMAGE RETRIEVAL

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Introduction

Tikhonov's method is applied for the solution of image deconvolution inverse problem in two-dimensional case. It appeared possible to improve the resolution beyond the aperture limit. The case of many-beam synthetic aperture radiometers (SAR) measurements is also considered. The method is applied to improve the resolution of radiobrightness image of oil spills on lakes. Data have been obtained from helicopter-borne radiometer measurements of thermal radio emission. The problem of the retrieval of true radiobrightness distribution by two-dimensional distribution of measured antenna temperature is very important in radioasrtonomy as well as in remote sensing, especially in the case of SAR measurements. The antenna temperature distribution is a two-dimensional convolution of radiobrightness distribution and antenna pattern as a kernel of the integral. If the kernel is a known function, it is possible to formulate the deconvolution inverse problem to retrieve the true radiobrightness image by measured antenna temperature distribution.

The deconvolution inverse problem consists of the solution of Fredholm integral equation of the 1-st kind, and it is well known that this problem is ill-posed. To solve such a problem it is necessary to use additional (*a priori*) information about the exact solution. This information determines a regularization method. There are various approaches: statistical (maximum entropy) [1], iterative [2], singular systems analysis [3]. In the present paper Tikhonov's method of generalized discrepancy is applied, which uses the common information about the exact solution as a function [4]. It is supposed that the exact solution belongs to the set of square-integrable functions with square-integrable derivatives. The results of numerical simulation give us the retrieval accuracy at various levels of the refraction error.

Problem formulation

The relationship between antenna temperature and radiobrightness distribution can be written as

$$\mathbf{K_h} T_b = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_h(x-s, y-t)\, T_b(s,t)\, ds\, dt = T_a^{\delta}(x,y), \qquad (1)$$

where $K_h(w,W)$ is the antenna pattern (kernel), T_a^{δ} is measured antenna temperature, and $T_b(s,t)$ is the radiobrightness to be found. The measure δ of the error of measured antenna temperature (measurement errors) and measure h of the kernel error satisfy to

$$\left\| T_a^{\delta} - T_a \right\|_{L_2} \le \delta, \qquad \left\| \mathbf{K} - \mathbf{K}_h \right\|_{W_2^2 \to L_2} \le h, \qquad (2)$$

where T_a corresponds to the exact solution. According to generalized discrepancy method, the approximate solution T_b^{α} of equation (1) minimizes the generalized discrepancy functional

$$\mathbf{M}_{\alpha}[T_b] = \left\| \mathbf{K}_h T_b - T_a^{\delta} \right\|_{L_2}^2 + \alpha \left\| T_b \right\|_{W_2^2}^2 \qquad (3)$$

at the condition

$$\left\| \mathbf{K}_h T_b^{\alpha} - T_a^{\delta} \right\|_{L_2}^2 = (\delta + h \left\| T_b^{\alpha} \right\|)^2 .$$

The use of Fourier transform permits to obtain the exact solution of the convolution-type equations in the form:

$$T_b^{\alpha}(s,t) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\tilde{K}_h^{*}(\omega,\Omega)\, \tilde{T}_a^{\delta *}(\omega,\Omega)\, e^{i\omega s + i\Omega t}\, d\omega d\Omega}{L(\omega,\Omega) + a[1 + (\omega^2 + \Omega^2)^2]} \qquad (4)$$

where

$$\tilde{K}_h^{*}(\omega,\Omega) = \tilde{K}_h^{*}(-\omega,-\Omega),$$

$$L(\omega,\Omega) = \left| \tilde{K}_h(\omega,\Omega) \right|^2,$$

$$\tilde{T}_a^{\delta}(\omega,\Omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} T_a^{\delta}(x,y)\, e^{i\omega x - i\Omega y}\, dx dy, \qquad (5)$$

$$\tilde{K}_h(\omega,\Omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_h(u,w)\, e^{i\omega u - i\Omega w}\, du dw. \qquad (6)$$

The main preference of Tikhonov's method consists of the uniform convergence of the retrieval error to zero at mean square convergence of measurement errors. The closed form of the exact solution (4) and the possibility to use Fast Fourier Transform (FFT) codes permits to make very efficient algorithm. Its accuracy has been investigated in numerical simulations. It appeared possible to retrieve images beyond the aperture resolution limit.

Results of numerical simulations

The most interesting results have been obtained in the case of image retrieval by multi-beam SAR data. For more compact presentation, an example was selected, in which two-beam antenna pattern $K_h(w,W)$ coincides with two-modal initial radiobrightness distribution $T_b(s,t)$. (see, in Fig.1). In the Fig.2 the corresponding distribution of antenna temperature distribution (observed image) $T_a(x,y)$ from (1) is given. It is possible to see that the two maxima of true T_b distribution are indistinguishable in observed image T_a.

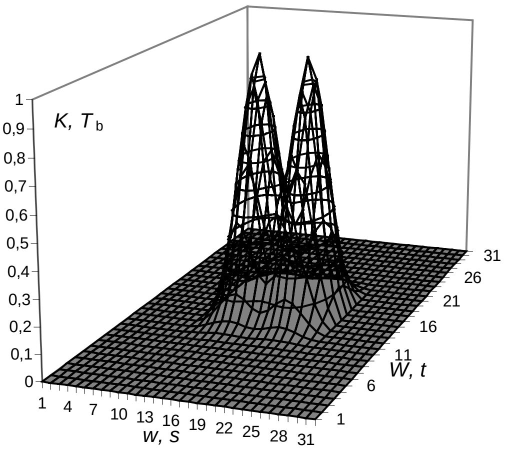

Fig.1. Antenna pattern and initial radiobrightness distribution, in conventional units

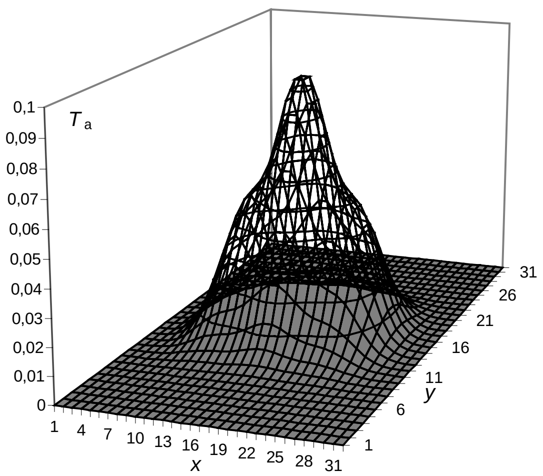

Fig.2. The corresponding observed image.

The approximate solution (4) is shown in Fig.3,4 at two different values of simulated measurement error δ (with respect to T_a, in integral L_2 - space). The Tikhonov's method of image retrieval permits to distinguish two-modal structure of initial distribution even at comparatively low measurement accuracy.

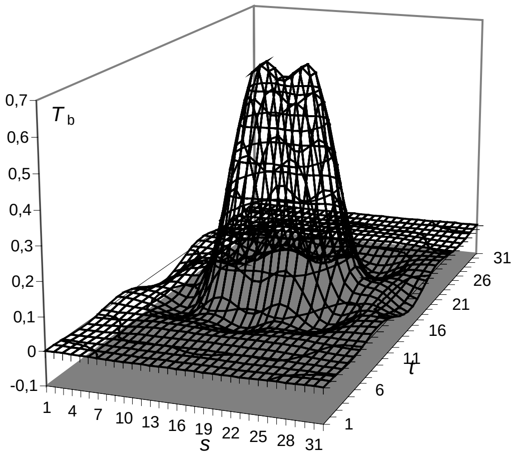

Fig.3. Retrieved image at error level 1%.

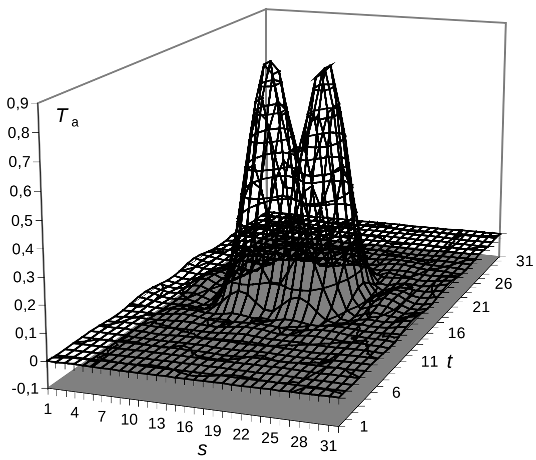

Fig.4. Retrieved image at error level 0.01%.

Along with error decrease, the retrieval error converges to zero, and it appears possible to retrieve true radiobrightness distribution beyond the aperture resolution limit even in the case of many-beam SAR measurements. One can see that the initial distribution in Fig.1 and retrieved image in Fig.4 are very similar.

There is an opinion that it is impossible to resolve two point sources if they are inside diffraction resolution limit because the spectra of real antennas are bounded and it is impossible to resolve frequencies beyond maximum frequency. But one has no periodic structure in the case, for example, of two point sources, and it is possible to use known information about the exact solution including its specific character at high frequencies.

Experimental results

The method has been applied to improve the resolution of radiobrightness image in the case of helicopter-borne radiometer measurements of thermal radio emission cm of oil spills on lakes at wavelength 3 cm [5]. The size of antenna pattern footprint was about 20 m. It was impossible to improve the resolution using large apertures because montage conditions on helicopter, and it was impossible to improve it by means of measurements at lower heights because of influence of propeller wind on oil spills.

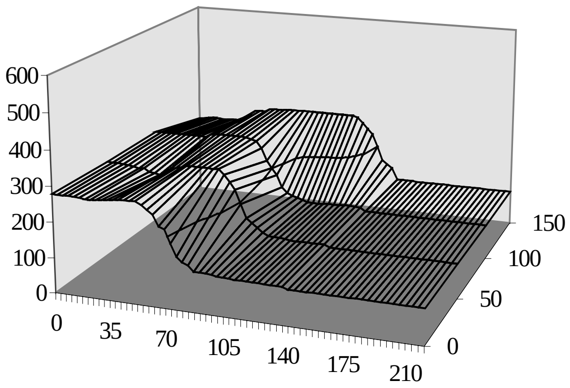

Fig.5. Measured radiobrightness distribution.

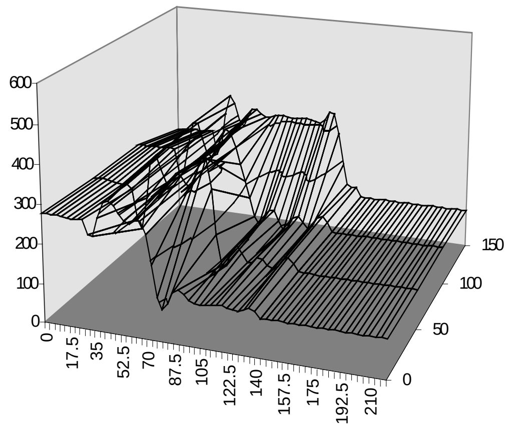

Fig.6. Retrieved radiobrightness distribution.

The only possibility was to solve inverse problem using high sensitivity of radiometer (about 0.1 K). One can see that the retrieved radiobrightness distribution contains more thin details of structure, and these details have been observed visually as thin folds across wind direction.

References

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