DIRECT AND INVERSE PROBLEMS OF ASTRONOMICAL REFRACTION

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Abstract. Physical and mathematical aspects have been investigated fox the formulation and solution of the inverse problem of the astronomical refraction –retrieval of the refractive index, pressure and temperature profiles from ground-based measurements of optical refraction at positive elevation angles, Accuracy estimations have been obtained for the vertical profile retrieval of the atmosphere parameters as a function of the measurement errors. A possibility is shown fox the retrieval of profiles with the temperature inversions.

At present, of great interest are possibilities of the remote sensing of the atmosphere parameters from the refraction measurements. The present paper considers a problem of the vertical profile retrieval of the refraction index, pressure and temperature ox the atmosphere parameters from measurements of the astronomical refraction, i.e. from ground measurements at positive elevation angles θ. Such a problem is reduced to the integral Fredholm equation of the first kind. Making integration by parts of the expression for the refraction angle

$$\varepsilon(p_\theta) = -p_\theta \int_{p_0}^{\infty} \frac{d\ln n}{dp} \frac{dp}{\sqrt{p^2 - p_\theta^2}} \; ; \; p = nr \; ; \; p_\theta = n_0 r_0 \cos\theta \quad (1)$$

($n_0 = n(r_0)$ is the near-ground value of the refraction index, r the distance from the Earth's center},

we obtain the equation

$$\int_{p_0}^{p_H} N(p) \frac{p p_\theta}{(p^2 - p_\theta^2)^{3/2}} \, dp = \tilde{\varepsilon}(p) , \qquad (2)$$

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where $N = 10^6 (n-1)$ is the refraction index, $\tilde{\varepsilon}$ is the modified right-hand side, p_H corresponds to a sufficient large height H. One may determine pressure (P) and temperature (T) vertical profiles of the atmosphere making measurements in the optical wave range, knowing N(p) and using the hydrostatic equation and perfect gas law.

It is known, that without as essential *a priori* information the solution of (2) is, generally, unsolved problem (Tikhonov, et al., 1983). This information is provided by the specific character of the concrete inverse problem. To solve the problem considered it is possible to use the information on N(p) belongings to the compact set of monotonous functions as well as the information on inter-level covariance relations (Tikhonov et al., 1983, Turchin, et al., 1970).

The solution of the inverse problem is rather difficult, if a number of questions associated with the solution of the direct problem is not answered, such as the questions on the direct value of natural variations of the refraction σ_ε as a function of the elevation angle θ, and also on the value of variation σ_{ε/N_0} at the fixed ground refraction index N_0 (the accuracy of the "Laplace theorem"). These values determine demands to the measurement accuracy and angle range, since the measurement accuracy must be sufficient for the measurements of the refraction variations related with natural variability of the profile N(p). The mean values of refraction <ε >, its variations σ_ε , variations of refraction σ_{ε/N_0} at the fixed N_0 as a function of θ have been calculated for a large ensemble of meteorological data, (see the Table)

θ	I"	I′	10'	30′	I°	2^0	5°	4^0	5°
<ε >	41'	41'	38'	33'	27'	20'	15'	12'	10'
σ_ε	4' 15"	4' 06"	3' 50"	3' 18"	2' 44"	1' 59"	1' 33"	1' 15"	2' 02"
σ_{ε/N_0}	43"	43"	37"	23"	12"	3.6"	1.6"	0.85"	0.51"

It is seen, that the specific character of N distribution is manifested in noticeable variations of the refraction only at reasonable small elevation angles. Thus, with a modern accuracy of measurements of the optical refraction at small elevation angles δε= 1÷10″, the informative angle range is found at $\theta<2\div4^0$. Numerical modeling has been made for the solution of the inverse problem by the method of solution on compact set of the monotonous functions and by the method of statistical regularization, as well as the statistical analysis of the restoration errors for large ensembles.

It has been found that with the realizing measurement accuracies (1-10″) the method of the statistical regularization gives more accurate results, while the method of solution on the compact set is preferable at higher accuracies. It is stated that the refraction measurements with the errors δε<10″ permits one to retrieve the distribution N(p) with an accuracy markedly exceeding that of the statistical optimal extrapolation over N_0. The quantity of the refraction measurements in the informative angle has a weak influence on the solution accuracy, that is explained by a strong correlation of the refraction values at close angles.

Computing results of rms errors of retrieval show that with the meusurement error δε=1" the accuracy of the refraction index determination amounts δN<15 N- units, the accuracy of the pressure retrieval is δP < 2 mbar and the temperature retrieval accuracy is δT = 1÷2 K in the height interval up to 8 km. For δε =10" such accuracies take place up to ~ 4 km heights. Fig.1 shows an example of the temperature profile retrieval in the numerical experiment at the modeled measurement error δε =10" by the method of statistical regularization. It is seen, that the basic peculiarities of the profile are retrieved, in particular, a strong temperature inversion in lower layers of the atmosphere.

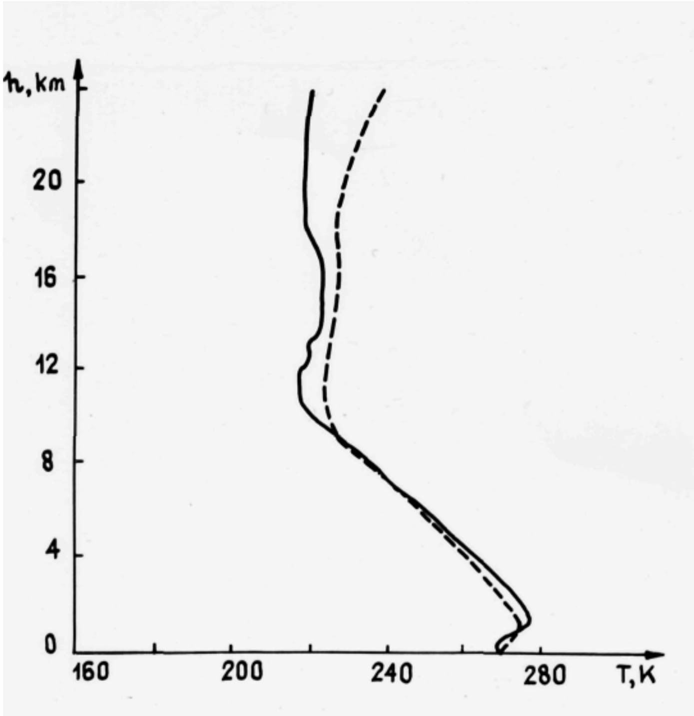

Fig.1. Restoration of the temperature profile with a strong inversion in the boundary layer by the method of statistical regularization for δε =10". A solid line is the initial profile, a dotted line is the retrieved one.

Thus, with the state of art measurement accuracies of the refraction, the meteorological parameter height profiles may be retrieved by the solution of the refraction inverse problem with an accuracy being compared with the estimations of the retrieved accuracy of meteorological parameters by the methods of ground-based microwave radiometry. This shows the perspectives of the considered refractometric method for the ground-based remote sensing of the atmosphere.

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