NEAR-FIELD MICROWAVE TOMOGRAPHY

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Abstract

A method of the near-field scanning tomography is developed for the microwave diagnostics of the 3D subsurface structure of permittivity. This method uses data of 2D scanning over lateral co-ordinates above the ground surface with the dipole emitter-receiver system. Measurements at several wavelengths provide the necessary depth sensitivity. The approach to the near-field inverse scattering problem based on the plane wave decomposition of proper Green functions is in use here with taking into account transfer functions of antennas designed for the microwave subsurface sounding. Results are presented for the tomography of 3D permittivity distribution originated by the ice sample buried in the sand.

Keywords: dipole antennas, microwave subsurface sounding, near-field scanning tomography, permittivity

1. Introduction

Methods of active and passive electromagnetic subsurface sounding make possible to determine the permittivity structure and temperature distribution inside the studied media [1]. The two-dimensional (2D) scanning above the surface is used typically to detect and visualize (obtain the 2D image) of buried objects. No processing is needed for this simple application.

A more complicated task is to retrieve one-dimensional distributions (profiles) of parameters. In this case the received signal should be depth-sensitive and measured in dependence on the corresponding parameter (depth-of-formation parameter). It can be the pulse delay time or frequency (at coherent sounding). Pulse methods are effective for multilayered structures; coherent methods are more suitable for continuous profiles. Profiles are obtained from solutions of corresponding integral equations; methods of regularization are in use to solve these ill-posed inverse problems [1].

Tomography methods (retrieval of 3D structures) lead to much more complicated problems. The 2D lateral scanning should be carried out in dependence on a depth-of-formation parameter; the inverse problem is reduced to the solution of the 3D integral equations of the 1-st kind with the 6D kernel. It is clear that straightforward methods of solution lead to hard restrictions on achievable dimensions of the region of retrieval and, hence, to the limitation of the achievable resolution. For far-field measurements there is also the known Rayleigh limitation of resolution.

In considered here method of coherent microwave near-field tomography of the subsurface permittivity, above-mentioned difficulties have been surmounted. Measurements of the scattered signal for an ice piece buried in the sand and serving as a target have been carried out using experimental set-up including vector network analyzer Agilent E5071B, two identical bow-tie antennas [2] in bi-static configuration, operating in the frequency range of 1.7 to 7.0 GHz and sandbox.

The frequency serves here as the depth-of-formation parameter that determines the depth sensitivity. In order to obtain the 3D information about the target, a C-scan has to be obtained. This is achieved by collecting a series of A-scans (801 points over the frequency range) on a horizontal survey lines (11×15 measuring points through 2 cm) over the sandbox surface.

To retrieve the 3D distribution of the permittivity, we use the general approach to the near-field scanning tomography [3] and its application to the near-field electromagnetic scattering [4] based on 2D lateral plane wave deconvolution of corresponding Green functions that reduces the initial 3D integral equation of the perturbation theory to the one-dimensional equation relative to the depth profile of the lateral spectrum of permittivity. This approach overcomes problems of the solution of 3D integral equations and leads to a high-performance mathematically consistent algorithm based on the Tikhonov's method of generalized discrepancy.

2. Theory

The proposed measurements scheme with the fixed emitter-receiver relative position has an important advantage – variations of the received signal (complex amplitudes) are related to the scattered field component only. So we use in the analysis the difference Δs between the signal above the sample region and outside it. This difference is a convolution of the apparatus function and the scattered field distribution $\mathbf{E}_1(x,y,\omega)$, so the lateral spectrum (2D Fourier transform over x, y) of Δs is:

$$\Delta s(k_x,k_y,\omega)=4\pi^2\mathbf{F}(k_x,k_y,\omega)\mathbf{E}_1(k_x,k_y,\omega)\,. \quad (1)$$

where $\omega=2\pi f$. Then, to solve the tomography problem, we obtain the spectrum of $\mathbf{E_1}$.

Let us consider the general case of multilayered dielectric medium following [4]. If the scattering region is embedded in *i*-th layer with $\varepsilon_0=\varepsilon_{0i}$, the complex permittivity can be written as $\varepsilon(\mathbf{r})=\varepsilon_{0i}+\varepsilon_1(\mathbf{r})$. The reference (unperturbed) field $\mathbf{E_0}(\mathbf{r})$ that is used in sounding is determined by the proper Green tensor:

$$\mathbf{E}_0(\mathbf{r})=\int\limits_V \mathbf{j}(\mathbf{r}')\ddot{\mathbf{G}}(x-x',y-y',z,z')d\mathbf{r}'\,, \quad (2)$$

where $\mathbf{j}(\mathbf{r})$ is the current distribution in the source antenna. Using the plane wave decomposition of (2) over *x* and *y*, it is possible to obtain $\mathbf{E_0}(\mathbf{r})$ in any layer. Above the scattering region, the electric field $\mathbf{E}(\mathbf{r})$ is obtained from the Fredholm equation of the 2-nd kind

$$\mathbf{E}(\mathbf{r})=\mathbf{E_0}(\mathbf{r})+\frac{i\omega}{4\pi}\int\limits_V \varepsilon_1(\mathbf{r}')\ddot{\mathbf{G}}(\mathbf{r},\mathbf{r}')\mathbf{E}(\mathbf{r}')d\mathbf{r}'\,, \quad (3)$$

that has the solution

$$\mathbf{E}(\mathbf{r})=\mathbf{E_0}(\mathbf{r})+\frac{i\omega}{4\pi}\int\limits_V \ddot{\mathbf{R}}(\mathbf{r},\mathbf{r}')\mathbf{E_0}(\mathbf{r}')d\mathbf{r}'\,. \quad (4)$$

The resolvent **R** is determined by the known Neumann series. Equations (2), (4) solve the direct problem of electrodynamics. The proper way to begin the study is to use the Born approximation, where the scattered field $\mathbf{E_1}(\mathbf{r})$ (the second term in (4)) is determined as

$$\mathbf{E}_1(\mathbf{r})=\frac{i\omega}{4\pi}\int\limits_V \varepsilon_1(\mathbf{r}')\ddot{\mathbf{G}}(x-x',y-y',z,z')\mathbf{E_0}(\mathbf{r}')d\mathbf{r}' \quad (5)$$

The equation (5) isn't, in general, a convolution equation over lateral co-ordinates relative to ε_1, but we use the measurement scheme proposed in [4] that makes it possible to reduce (5) to the convolution equation. In this scheme the structure of sounding field is invariable relative to the receiver position because we fix the source-receiver vector $\boldsymbol{\delta}\mathbf{r}$. In this case the reference field (2) in (5) can be written as

$$\mathbf{E}_0(\mathbf{r},\mathbf{r}',\boldsymbol{\delta}\mathbf{r})=\int\limits_{V'} \mathbf{j}(\mathbf{r}''-\mathbf{r}-\boldsymbol{\delta}\mathbf{r})\ddot{\mathbf{G}}^{kl}(x'-x'',y'-y'',z',z'')d\mathbf{r}'' \quad (6)$$

This representation reduces the 3D integral equation (5) to the convolution equation over lateral co-ordinates and, after the 2D Fourier transform – to the desired one-dimensional integral equation relative to the depth profile of the lateral spectrum of ε_1:

$$\begin{aligned}\Delta s(k_x,k_y,\omega)&=4\pi^2F_i(k_x,k_y,\omega)E_{1i}(k_x,k_y,\omega)=\\&=\int\limits_{z'}\varepsilon_1(k_x,k_y,z')\cdot\\&\cdot i4\pi^3\omega\,F_i(k_x,k_y,\omega)\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}e^{i(k_x+\kappa_x)\delta x+i(k_y+\kappa_y)\delta y}d\kappa_x d\kappa_y\cdot\\&\cdot\{\int\limits_{z''}[j_i(-k_x-\kappa_x,-k_y-\kappa_y,z''-z-\delta z)\cdot\\&\cdot g_{ij}^{kl}(k_x+\kappa_x,k_y+\kappa_y,z',z'',\omega)]g_{ji}^{lk}(\kappa_x,\kappa_y,z,z',\omega)dz''\}dz'\end{aligned} \quad (7)$$

for each pair of lateral spectral components. Here g_{ji}^{lk} are components of the lateral 2D Fourier transform of the Green function **G** that determine the contribution of the j^{th} component of the source to the i^{th} component of the field at the receiver position in the k^{th} layer and the position of the sounded region in the l^{th} layer. Also, we have taken into account that in our measurements the source and the receiver have the same (up to an unessential constant multiplier) transfer function ($F_i(k_x,k_y,\omega)=j_i(k_x,k_y,\omega)$). The effect of the spectrum shift in (7) makes it possible to realize the subwavelength resolution at near-field measurements.

The equation (7) is a Fredholm integral equation of the 1-st kind with the kernel that depends on the frequency (depth-of-formation parameter). To retrieve these depth profiles, the mathematically consistent algorithm [4] based on the Tikhonov's method of generalized discrepancy has been applied. Finally, when the spectrum is retrieved, the desired 3D structure of permittivity is obtained by the 2D inverse Fourier transform:

$$\varepsilon(x,y,z)=\iint\varepsilon(\kappa_x,\kappa_y,z)\exp(i\kappa_x x+i\kappa_y y)d\kappa_x d\kappa_y\,. \quad (8)$$

It is possible to generalize the solution beyond the Born approximation, using (6) and (7) in the equation of perturbation theory (3) [4].

3. MULTIFREQUENCY MICROWAVE NEAR-FIELD SCANNING TOMOGRAPHY

Using the equation (7) for the 2-layrered media (a half space with an inhomogeneous region), we have realized the method of multifrequency tomography based on the 2D lateral scanning of $\Delta s(x,y,f)$ in the frequency band of 1.7 to 7.0GHz. In following analysis, data have been used for only six chosen frequencies: 1.7, 2.76, 3.82, 4.88, 5.94, 7.0 GHz. Antennas (two identical bow-tie transmitting and receiving antennas with the length of arms 3.8 cm and the width of 5.4 cm, placed in the *y*-direction; distance between antenna centers of 7.5 cm) were scanning together in the rectangle *x*-*y* area. The target with sizes 10×10×4 cm has been buried in the sand at the depth $z = -9$ cm.

In Fig.1 the scanning results at the frequency f_2 = 2.76 GHz (wavelength 10.83 cm) are shown.

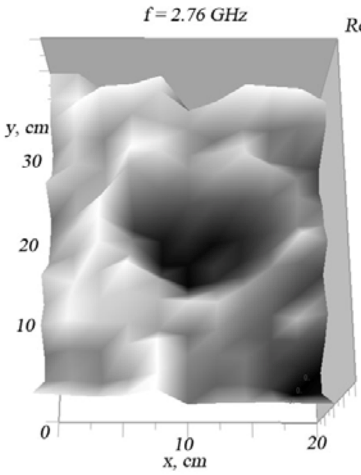

Fig. 1. Two-dimensional signal distribution at the frequency 2.76 GHz – result of the scanning above the region with the buried ice target.

One can see the microwave image of the buried ice target. This result shows the sensitivity of measurements to the subsurface inhomogeneous region. Also, it gives us the estimation of the relative error level (about 0.2) that determines the regularization parameter of the Tikhonov's method in the processing.

Results of 2D scanning at 6 frequencies have been used to solve the integral equation (7) relative to the depth profile of the lateral spectrum of permittivity perturbation $\varepsilon_1(k_x, k_y, z)$ for each pair of spectral components. Then, from (8), we have the desired 3D structure (tomogram) of $\varepsilon_1(x,y,z)$. In Fig.2 the vertical section of the retrieved 3D permittivity is presented.

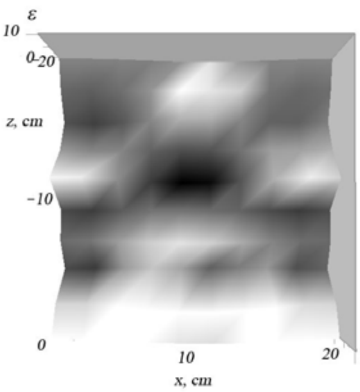

Fig. 2. Vertical section of the retrieved 3D permittivity (tomogram) $\varepsilon_1(x,z)$ at $y = 16$ cm.

Results are in a good enough correspondence with the real position and form of the ice sample and with permittivity variations related with the ice in the sand ($\varepsilon_{ice} = 2$; $\varepsilon_{sand} = 4.3$). Horizontal section of the retrieved permittivity tomogram at the depth of the buried target is shown in Fig.3.

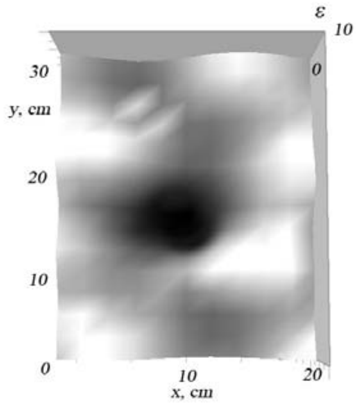

Fig. 3. Horizontal section of the retrieved 3D permittivity (tomogram) $\varepsilon_1(x,y)$ at $z = -9$ cm.

Again, taking into account a high enough level of measurement errors, results are in a reasonable correspondence with the real position and form of the ice sample.

In conclusion, we have applied a new method of near-field multifrequency coherent tomography in the microwave range to retrieve the subsurface 3D structure of permittivity. Our first results show the feasibility of this method.

4. Permission to Publish

The authors are responsible for all material contained in the manuscript they submit.

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