MULTIFREQUENCY DIAGNOSTICS OF INHOMOGENEOUS MEDIA

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Abstract

Method of electromagnetic scanning tomography and holography of subsurface objects based on the solution of inverse scattering problem for the pseudopulse synthesized by multifrequency measurements of the scattered field is studied. Results of its applications to diagnostics of various subsurface inhomogeneities are demonstrated.

Keywords: electromagnetic sounding, inhomogeneous media, complex permittivity, tomography, holography, inverse scattering problem.

1. INTRODUCTION

The method of scanning electromagnetic tomography proposed in [1] and developed [2], where the initial 3D problem has been reduced to the multiple solution of one-dimensional problem for the depth profile of lateral spectra of subsurface inhomogeneities, has been realized in [3] as the method of multifrequency microwave subsurface tomography and holography. In the latter paper, to discern the target signal from the strong surface scattering, the multifrequency problem has been transformed to that for synthesized pseudopulse. Here we demonstrate further results of this study.

2. THEORY

Following [1-3], let us consider a scattering region with the complex permittivity $\varepsilon(\mathbf{r}) = \varepsilon_0 + \varepsilon_1(\mathbf{r})$ that is embedded in a half-space $z \le 0$ with $\varepsilon = \varepsilon_0$. The total field at a frequency ω is a sum of probing and scattered field components $\mathbf{E}(x,y,\omega) = \mathbf{E}_0(x,y,\omega) + \mathbf{E}_1(x,y,\omega)$. For the scheme of measurements with the fixed source-receiver vector $\boldsymbol{\delta}\mathbf{r}$, when the structure of the probing field is invariable relative to the receiver position, it is possible to express the *k*-space spectrum (2D inverse Fourier transform over *x* and *y*) of the scattered field in frameworks of the Born approximation as a convolution over lateral co-ordinates [2,3]. Variations of complex amplitudes of the received signal *s* are expressed by the convolution of the instrument function $\mathbf{F}$ of the receiver and the scattered field $\mathbf{E}_1$:

$$s(\mathbf{r}_r) = \int \mathbf{E}_1(\mathbf{r}')\mathbf{F}(x_r - x', y_r - y', z_r, z')dx'dy'dz', \quad (1)$$

where $\mathbf{r}_r$ is the vector determining the receiver position. From (1) and (2), the transversal spectrum of measured signal variations is obtained as:

$$s(k_x,k_y,\omega) = \int_{z'} \varepsilon_1(k_x,k_y,z')K(k_x,k_y,z',\omega)dz'.$$

(2) This equation has been used in our algorithm of the microwave subsurface tomography [2]; but it was difficult to recognize sounded subsurface objects on the measured image of $s(x,y,\omega)$ against the noise produced by the surface scattering. However, we have found that it is possible to obtain much better images of subsurface targets, using the transformation of multifrequency data to synthesized pulse. It suggests that we make similar transformations in equation (2) [4]:

$$s(k_x,k_y,t) = \int_0^\infty s(k_x,k_y,\omega)\exp(i\omega t)d\omega, \quad (3)$$

that can be represented in dependence on the effective depth parameter z_s according $s(k_x,k_y,z_s) = s(k_x,k_y,t = 2z_s \operatorname{Re}\sqrt{\varepsilon_0}/c)$ that leads to the new equation that relates the complex permittivity spectrum to the complex-valued synthesized pulse of the signal lateral spectrum:

$$s(k_x,k_y,z_s) = \int_{z'} \varepsilon_1(k_x,k_y,z')K(k_x,k_y,z',z_s)dz' \quad (4)$$

$$K(k_x,k_y,z',z_s) = K(k_x,k_y,z',t = 2z_s \operatorname{Re}\sqrt{\varepsilon_0}/c),$$

$$K(k_x,k_y,z',t) = \int_0^\infty K(k_x,k_y,z',\omega)\exp(i\omega t)d\omega.$$

The depth dependence of $K(k_x,k_y,z',z_s)$ in (4) has maxima at depths z' that are placed deeper with the increase of $|z_s|$ – unlike the kernel of (2) that has maxima near the surface. It can explain the observed depth selectivity and resolution of such pseudopulse images as well as the suppression of the noise related to the surface scattering. Also, it is well-known that kernels with maxima, such as in (4), provide better solution results in comparison with the exponential kernel of the initial equation (2).

To solve the Fredholm integral equation, the algorithm based on the generalized discrepancy

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principle in the complex Hilbert space W_2^1 [2] has been applied here to retrieve tomography images of subsurface inhomogeneities with the complex-valued distribution of permittivity. From the solution of (4), the desired 3D structure of permittivity (tomogram) is obtained by 2D inverse Fourier transform:

$$\varepsilon_1(x,y,z) = \iint \varepsilon_1(\kappa_x,\kappa_y,z)\exp(i\kappa_x x + i\kappa_y y)d\kappa_x d\kappa_y \ . \quad (5)$$

Note that in practice subsurface targets mostly have a homogeneous internal structure. When it is known *a priori* that the permittivity of a target $\varepsilon_1^0 = const$, the tomography problem can be reduced to the problem of the target shape retrieval, i.e. to the problem of the computer holography [3]. For that, making the inverse Fourier transform of (3) over k_y, obtain the complex-value transcendent equation:

$$\varepsilon_1(k_x,y',z) = \int_{-\infty}^{\infty} \varepsilon_1(k_x,k_y,z)\exp(ik_y y')dk_y = \left(\frac{\varepsilon_1^0}{2\pi i k_x} e^{-ik_x x_1(y',z)} - e^{-ik_x x_2(y',z)}\right) , \quad (6)$$

that is equivalent to the system of two real equations. The desired shape of the target expressed by two functions $x_1(y,z)$, $x_2(y,z)$ is obtained from this equation, using the solution $\varepsilon_1(k_x,k_y,z)$ of (3). This equation is overdetermined: it can be solved at each value of k_x.

Similar methods can be developed in the much simpler problem of low-frequency diagnostics of the Earth crust conductivity subsurface inhomogeneities $\sigma_1(x,y,z)$ that is applied in the geomagnetic exploration, where $\varepsilon = \varepsilon' + i\varepsilon'' \approx i4\pi\sigma/\omega$. Using the solution of the corresponding tomography problem as described in [1], it is possible to obtain holography images of 3D intrusions of conductivity from

$$\sigma_1(k_x,y',z) = \left(\frac{\sigma_1^0}{2\pi i k_x} e^{-ik_x x_1(y',z)} - e^{-ik_x x_2(y',z)}\right). \quad (7)$$

3. EXPERIMENT

This theory has been realized in algorithms of multifrequency microwave tomography and holography and studied in experiments [3]. Measurements of signal complex amplitudes for 801 frequencies in the range 1.7 – 7.0 GHz obtained by 2D lateral scanning have been used in analysis. The source-receiver system based on the vector network analyzer Agilent E5071B includes two identical transmitting and receiving planar bow-tie antennas (with length of arms 3.8 cm and width of 5.4 cm, placed in *y*-direction; the fixed distance between centers of antennas was 7.5 cm) in bistatic configuration placed in *y*-direction. They were scanning together in the rectangle *x-y* area above buried targets. For bow-tie transmitting and receiving antennas, used in measurements. The current distribution on the antenna and its spatial spectrum are shown in Fig. 1 for the lowest frequency of analysis.

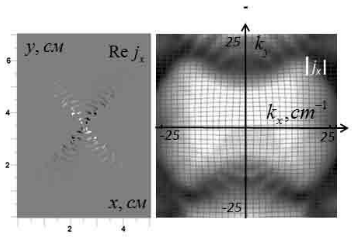

Fig. 1. Left, the current distribution on the bow-tie antenna at 1.7 GHz; right, the lateral spectrum of this current distribution.

One can see that the current distribution is quite sharply localized at the center of antenna's surface, so it has a very broad spatial spectrum, where components with $k_x, k_y > 2\pi/\lambda$ dominate, and, hence, they form a broad near-field spectrum of the signal. It makes possible to realize the subwavelength resolution of targets in proposed tomography and holography.

In Figs. 2-3, results of our experimental study of the depth sensitivity of the proposed tomography method for thin parallelepiped foam samples with sizes 4 × 4 × 1 cm^3 buried in the sandy ground (the same, as in [3]) are demonstrated at depth of targets z_t = –1 cm and z_t = –7 cm.

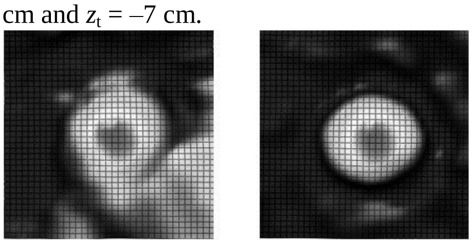

Fig. 2. Images of the signal amplitude measured in the range of scanning 20×20 cm^2. Left, for the foam target at z_t = –1 cm at z_s = –1 cm; right, for the target at z_t = –7 cm at z_s = –7 cm.

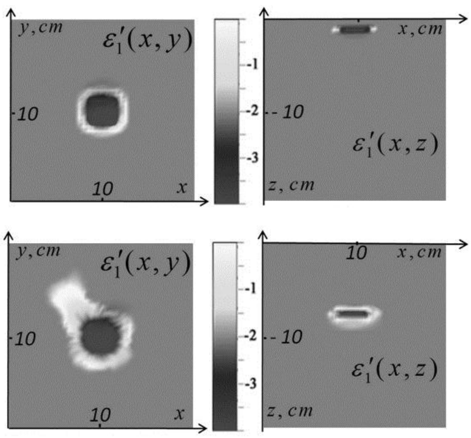

Fig. 3. Results of subsurface tomography of target at depth z_t = –1 cm (upper row) and z_t = –7 cm (lower row). Left, horizontal sections at depth z = –1.5 cm (upper row), or at z = –1.5 cm (lower row); right, vertical sections at y = 10 cm.

Permittivity images in Fig. 3 demonstrate high enough retrieval accuracy the inverse problem solution. These results have been used to obtain holography images of targets from the solution of (6) (see in Fig.4).

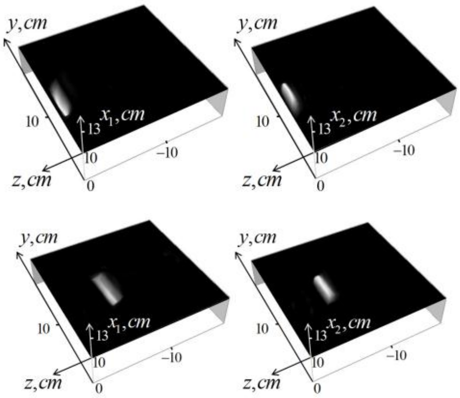

Fig. 4. Results of subsurface holography of target at depth z_t = –1 cm (upper row) and z_t = –7 cm (lower row) shown as functions $x_1(y,z)$ (left) and $x_2(y,z)$ (right).

In Fig. 4, one can see high enough retrieval accuracy of parameters, position and shape of targets. Taking into account signal wavelengths in the medium (2.2 – 9 cm), results demonstrate a subwavelength resolution of holography images.

Our study includes also biology targets in dielectric media with high values of permittivity, typical for living tissues. In Fig.5 measurement results for parallelepiped fat samples with sizes 4 × 4 × 1 cm^3 in the solution of sugar (weight ratio 53.9%) in water (45%), and citric acid as preservative with $\varepsilon = 41.0 + i17.4$ (at 1.7 GHz) are shown.

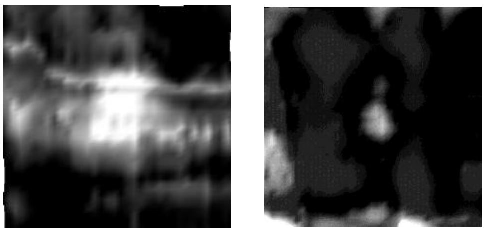

Fig. 5. Images of the signal amplitude measured in the range of scanning 20×20 cm^2. Left, for the fat target at z_t = –0.5 cm, z_s = –2.5 cm; right, for target at z_t =–1.5 cm, z_s = –2 cm.

From comparison of Fig. 2 and Fig.5 one can see that measurements in the latter case – for targets with $\varepsilon = 11.3 + i2.2$ in the highly reflective and strongly absorbing medium (skin-depth d_{sk} = 2.1 cm) – are much less sensitive.

To solve inverse scattering problems in such cases of high contrasts of permittivity, it is necessary to use corrections to Born approximations, proposed in [2,3]. More generous approach to non-linear inverse scattering problems based on the dual regularization method in the theory of non-linear ill-posed inverse problems that has been successfully applied in solution of one-dimensional problems [4-6] is now under development. A 3D algorithm of this method has been proposed in [7], and corresponding code parallelization is now under construction for calculations with powerful multiprocessor computers.

Similar measurements have been carried out with muscle tissue targets with $\varepsilon = 55.9 + i19.3$ that have much less permittivity contrast with environment. However, by now, it appeared impossible to discern the corresponding scattered signal.

4. PERMISSION TO PUBLISH

The authors are responsible for all material contained in the manuscript they submit and agree to the submission of the paper.

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