ON RETRIEVAL OF PERMITTIVITY PROFILE[1]

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ABSTRACT

New theoretical approach is proposed for solving inverse problem of dielectric permittivity (i.e. refractive index) retrieval by reflected field. It is based on approximate solution of an integral equation, obtained by combining the integral equation for the electromagnetic field inside the object with integral expression, which determines the diffracted field in the external region from the field inside it. The solution based on the Tichonoff's theory of ill-posed problems is proposed. Also a simple scheme for fast estimation of the object permittivity profile has been worked out.

1. INTRODUCTION

Retrieval of material parameters such as refractive index (i.e. dielectric permittivity) profile is important in optical devices and systems for testing fabricated devices in the view of complicated technology, which makes sometimes impossible an accurate realization of refractive index profiles, especially if the profiles are continuous. Fast methods of retrieval can be attractive for application in optimization design problems, as well as for monitoring measurements.We propose a method based on approximate solution of integral equation for external field in 1D, valid for rather thin objects. This equation solution is an expression for the external (reflected) field, determined after the internal one. It allows excluding the internal field from the initial internal equation of the diffraction problem, obtaining then an equation connecting the external field and the object parameters. The solution provides algorithms for the permittivity profile determination.

2. MATHEMATICAL BACKGROUND

For the plane harmonic incidence as $\vec{E}(t,x) = E(x)e^{i\omega t}\vec{e}_z$.

the wave equation $$\frac{d^2E(x)}{dx^2} + \frac{\omega^2}{c^2}\left(\frac{\varepsilon(x)-\varepsilon_0}{\varepsilon_0} + \cos^2\theta\right)E(x) = 0$$

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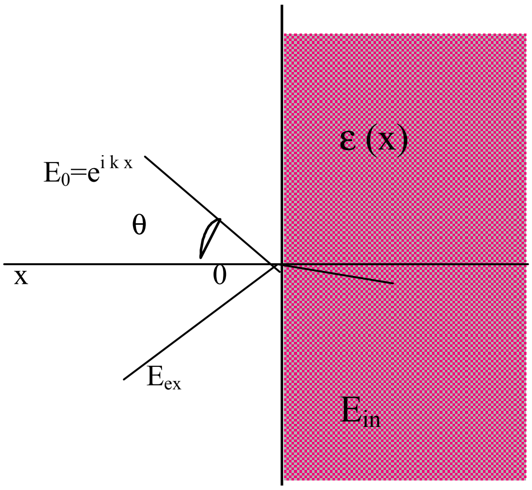

Fig.1. Problem geometry and notations

enables one to obtain the following integral equation for the field by Green's function approach:

$$E(x) = E_0(x) + \frac{\omega^2 i}{2kc^2}\int_0^\infty dx' \frac{\varepsilon(x') - \varepsilon_0}{\varepsilon_0} E(x')\, e^{ik|x-x'|},$$

where $k \equiv \frac{\omega}{c}\cos\theta$.

This equation has the following representation for the internal field for x>0 (see Fig.1 for the problem geometry and accepted notations):

$$E_{in}(x) = E_0(x) + \frac{\omega i}{2c\cos\theta}\left[\int_0^x dx' \frac{\varepsilon(x') - \varepsilon_0}{\varepsilon_0} E_{in}(x')\; e^{ik(x-x')} + \int_x^\infty dx' \frac{\varepsilon(x') - \varepsilon_0}{\varepsilon_0} E_{in}(x')\; e^{-ik(x-x')}\right]. \quad (1)$$

For the external field (x<0) it takes a form of expression for the external field, determined after the internal one:

$$E_{ex}(x) = E_0(x) + \frac{\omega i}{2c\cos\theta}\int_0^\infty dx' \frac{\varepsilon(x') - \varepsilon_0}{\varepsilon_0} E_{in}(x')\; e^{-ik(x-x')} \quad (2)$$

$E_{ex}(x) - E_0(x) = E_{ref}(x) = r\, e^{-ikx}$, where r is a complex-value reflection coefficient.

Incident field $E_0(x) = e^{ikx}$.

Combining expressions (1) and (2), the following integral equation for the internal field can be obtained:

$$E_{in}(x) = e^{ikx} + r\, e^{-ikx} - \frac{\omega}{c\cos\theta}\int_0^x dx' \frac{\varepsilon(x') - \varepsilon_0}{\varepsilon_0} \sin k(x-x') E_{in}(x') \quad (3)$$

Its solution can be obtained by convergent iterations, determining then the internal field after the reflected one. This solution substitution into the equation (2) yields the following integral equation for the permittivity profile:

(4)

$$r = \frac{1}{2i}\int_0^\infty dx'\, \tilde{\varepsilon}(x')\left(e^{ikx'} + re^{-ikx'}\right)\left\{e^{ikx'} + \int_{x'}^\infty dx''\, \tilde{\varepsilon}(x'') \sin k(x''-x') \cdot \left\{e^{ikx''} + \int_{x''}^\infty dx'''\left\{\tilde{\varepsilon}(x''') \sin k(x'''-x'')...\right\}\right\}\right\}$$

$$\tilde{\varepsilon} = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0\sqrt{\varepsilon_0}\cos\theta}.$$

3. SOLUTION OF INTEGRAL EQUATION

A very complicated nonlinear integral equation of the 1-st kind (4) could be solved using approach developed in [1] if the reflection coefficient r as a function of frequency ν or incident angle θ (or both) have been measured. It is possible to express (4) as

$$\int_0^\infty \tilde{\varepsilon}(x')K(\tilde{\varepsilon},x',\nu,\theta)dx' = r(\nu,\theta), \qquad (5)$$

where $K(\tilde{\varepsilon},x',\nu,\theta)$ is a nonlinear kernel of the integral equation (4). If there is the convergence, in the first guess of the algorithm proposed in [1] an arbitrary model profile could be used in the kernel. In the next iteration step the retrieved profile should be used. Each of iterations consists of the solution of Fredholm integral equation of the 1-st kind. It is a well-known ill-posed problem. To solve this problem it is necessary to use some additional *a priori* information about the exact solution. One of the most suitable methods is Tichonoff's method of generalized discrepancy [2], which implies that the exact solution belongs to the set of functions summable with a square that also have generalized derivatives also summable with a square.

In the case of 1D medium as multi-layered, the permittivity profile can be determined as $\varepsilon(x) = \sum_{j=1}^{N} \varepsilon_j \left(h(x-a_j) - h(x-a_{j+1})\right)$, where a_j and a_{j+1} are coordinates of the left and right boundaries of j^{th} layer, h is the Heviside's step function.

If we consider only the first term of the sum (one iteration in the integral equation solution) and assume that $\Delta\tilde{a}_j \equiv \tilde{a}_i - \tilde{a}_{i-1} << 1$ (where $\tilde{a}_j \equiv k_j a_j$ is the normalized width of the k-th sublayer), we obtain a very simple equation for the local permittivity values:

$$-0.5 \cdot C_1 \cdot \sum_{k=1}^{N} \tilde{\varepsilon}_k\, \Delta\tilde{a}_k + \sum_{k=1}^{N} \tilde{\varepsilon}_k\, \Delta\tilde{a}_k \left[C_2 \cdot \left(\sum_{l=1}^{k-1} \Delta\tilde{a}_l + \frac{\Delta\tilde{a}_k}{2} \right) + \right. \\ \left. + C_3 \cdot \left(\Delta\tilde{a}_k \sum_{l=1}^{k-1} \Delta\tilde{a}_l + 2 \sum_{l=1}^{k-1} \Delta\tilde{a}_l \sum_{j=1}^{l} \Delta\tilde{a}_{jl} \right) \right] = 1, \qquad (5)$$

where $\tilde{\varepsilon}_k = \dfrac{\varepsilon_k - \varepsilon_0}{\varepsilon_0\sqrt{\varepsilon_0}\cos\theta}$, $\Delta\tilde{a}_k = \dfrac{\Delta a_k \sqrt{\varepsilon_0}}{c}\cos\theta$ is normalized thickness of k^{th} sub-layer. The reflection coefficient and the incidence frequency are contained in the coefficients C: $C_1 = \omega \sin\varphi$, $C_2 = \omega^2 \cos\varphi$, $C_3 = \omega^3 \sin\varphi$. After solving this equation for different incident frequencies or different incidence angles the permittivity of each sub-layer can be obtained from the system of linear algebraic equations. Their number is equal to the number of the considered sub-layers, and so the computational procedure is very fast.

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