# Near-Field Inverse Scattering Problems in Noninvasive Diagnostics of Subsurface Dielectric Structures

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**ABSTRACT**
A review and last results in the region of subsurface near-field electromagnetic diagnostics of dielectric inhomogeneities are presented. Methods of nondestructive testing of 3D distributed and solid targets (computer tomography and holography, respectively) as well as methods to retrieve the subsurface profiles of one-dimensional inhomogeneities in various media are considered.
**Keywords**: electromagnetic inverse scattering problems, tomography, holography.

## 1. INTRODUCTION

During the last decade, the new approach to noninvasive electromagnetic diagnostics based on the solution of inverse scattering problems taking into account not only wave fields components, but also near-field components has been developed for various applications covering the range from nanometers (for inhomogeneities of X-ray microstructures) to kilometers (in magnetotelluric sounding of conductivity inhomogeneities in the Earth crust), where the possibility to realize a subwavelength resolution has been demonstrated. From the middle of the last century, inverse problems are widely used in remote sensing and non-destructive diagnostics [1]. They are typically ill-posed, and various regularization methods and solving algorithms have been proposed and applied [1,2]. Unfortunately, there are no universal methods to solve the considered here nonlinear ill-posed problems. Nevertheless, some algorithm based on especial schemes of measurements, suitable approximations, and new achievements in mathematical theory have been worked out and used in this new area [3-18].

## 2. THEORY

In a quite generous case of an inhomogeneity $\varepsilon_1(\mathbf{r})$ located in $l^{\text{th}}$ layer of a multilayer medium with permittivity $\varepsilon(\mathbf{r}) = \varepsilon_l + \varepsilon_1(\mathbf{r})$, the complex amplitudes of vectors of electrical and magnetic fields **E**, **H** [$\sim \exp(-i\omega t)$] measured in $k^{\text{th}}$ layer are determined by components $j_i^k$ of the source current distribution located in the same $k^{\text{th}}$ layer expressed as the sum of the probing and scattered fields [4,6]:

$$E_i^k(\mathbf{r}) = E_{0i}^k(\mathbf{r}) + E_{1i}^k(\mathbf{r}) = \int_{V'} j_j^k(\mathbf{r}')G_{ji}^{kl}(x-x', y-y', z, z')d\mathbf{r}' - \frac{i\omega}{4\pi}\int_{V'} \varepsilon_1(\mathbf{r}')E_j^l(\mathbf{r}')G_{ji}^{lk}(x-x', y-y', z, z')d\mathbf{r}', \qquad (1)$$

where $G_{ij}^{kl} = {}^{\parallel}G_{ij}^{kl} + {}^{\perp}G_{ij}^{kl}$, $G_{ji}^{lk} = {}^{\parallel}G_{ji}^{kl} + {}^{\perp}G_{ji}^{kl}$ are components of Green tensors that are sums of terms for TE ($\perp$) and TH($\parallel$) - polarizations. The convenient summation over repeated indices (*i, j = x, y, z*) is implied in (3-5).

The solution of (1) can be obtained iteratively, beginning with the Born approximation (the first term of Neumann series). The statement of inverse scattering problems can be based on equation (1) considered as a non-linear integral equation of the 1st kind with the 6D kernel. Like in the direct problem, there is the evident way to begin the solution using the Born approximation in the expression for scattered field in (1):

$$E_{1i}^k(\mathbf{r}) = -\frac{i\omega}{4\pi}\int_{V'} \varepsilon_1(\mathbf{r}')E_{0j}^l(\mathbf{r}')G_{ji}^{lk}(x-x', y-y', z, z')d\mathbf{r}'. \qquad (2)$$

The solution of this 3D equation leads to strong limitations of the grid size used at calculations and, hence, to limitations of the achievable resolution. However, some methods have been proposed, where (2) appeared as a convolution equation over *x* and *y*, and, hence, it could be reduced a one-dimensional integral equation by 2D Fourier transform over transversal co-ordinates (low-frequency sounding of earth crust [4,6], total-internal-reflection tomography [3]). In these methods, a single plane wave $E_0(\mathbf{r}') = f_0(\kappa_x, \kappa_y, z')\exp\{i\kappa_x x' + i\kappa_y y'\}$ is used as the probing field (at that, evanescent waves can be used to realize a subwavelength resolution [3]), and one obtains from (2) the integral equation for components of the transversal spectrum:

$$E_{1i}^k(k_x, k_y, z, \omega) = -i\omega\pi\int_{z'} \varepsilon_1(k_x - \kappa_x, k_y - \kappa_y) f_j(\kappa_x, \kappa_y, z')G_{ji}^{lk}(k_x, k_y, z, \omega, z')dz'. \qquad (3)$$

It should be multiple solved by data of multilevel, multifrequency or multi-angle measurements for each pare $k_x$, $k_y$. Another approach has been developed in [7,9] for the scheme of measurements with the fixed source-receiver vector $\boldsymbol{\delta}\mathbf{r}$, when it appeared possible to express the *k*-space spectrum (2D inverse Fourier transform over *x* and *y*) of the scattered field in $k^{\text{th}}$ layer in frameworks of the Born approximation as:

$$E_{1i}^{k}(k_x,k_y,\omega,z,\boldsymbol{\delta}\mathbf{r}) = -4\pi^3 i\omega\int_{z'}\varepsilon_1(k_x,k_y,z')\{\iint \mathrm{e}^{-i\kappa_x\delta x - i\kappa_y\delta y}$$
$$\times\int_{z''}[j_i(\kappa_x,\kappa_y,z''-z-\delta z,\omega)G_{ij}^{kl}(\kappa_x,\kappa_y,z'',z',\omega)]G_{ji}^{lk}(\kappa_x+k_x,\kappa_y+k_y,z',z,\omega)d\kappa_x d\kappa_y dz''\}dz', \quad (4)$$

where spectral components of Green tensors can be obtained explicitly [9] (for brevity, we use the same notations for Fourier-transforms of these parameters). Based on the solution of (4), algorithms of multilevel perfect lens tomography [8,9] and multifrequency microwave tomography [9] have been proposed. To apply this theory in microwave experiments, it is necessary to take into account that the received signal $s$ is the convolution of instrument function and scattered field. However it doesn't change the kind of the equation that is written as $s(k_x,k_y,\omega) = \int_{z'}\varepsilon_1(k_x,k_y,z')K(k_x,k_y,z',\omega)dz$ [13]. In experiments, we hit a problem related to the noise produced by the surface scattering. Nevertheless, this problem has been overcome using the transformation of multifrequency data in (4) to the synthesized pseudopulse $s(k_x,k_y,t) = \int_{\Delta\omega}s(k_x,k_y,\omega)\exp(i\omega t)d\omega$ with the change $s(x_r,y_r,z_s) = s(x_r,y_r,t=-2z_s\,\mathrm{Re}\sqrt{\varepsilon_0}/c)$ (where $z_s$ is the parameter of the effective depth of a scattering element), the integral equation is written as

$$s(k_x,k_y,z_s) = \int_{z'}\varepsilon_1(k_x,k_y,z')K_1(k_x,k_y,z',z_s)dz', \quad (5)$$

This transformation leads to the depth dependence of $K_1(k_x,k_y,z',z_s)$ with maxima that can explain the observed depth selectivity and resolution of pseudopulse images and provide better results in comparison with the exponential kernel of initial equation. From the solution of (5), the desired tomogram is obtained by the 2D inverse Fourier transform $\varepsilon_1(x,y,z) = \iint\varepsilon_1(k_x,k_y,z)\exp(ik_x x + ik_y y)dk_x dk_y$ . Also, an iteration algorithm has been proposed for the correction of the solution obtained in the Born approximation [9].

For targets with a homogeneous internal structure, it is possible to obtain their shape (i.e. to solve the problem of computer holography) as two functions $x_1(y,z)$, $x_2(y,z)$ in each section $z$ = *const* using the $k$-space solution of (5) (its inverse 1D Fourier transform $\varepsilon_1(k_x,y',z) = \int\varepsilon_1(k_x,k_y,z)\exp(ik_y y')dk_y$ ), from the equation:

$$\varepsilon_1(k_x,y,z) = \varepsilon_1^0/2\pi ik_x(e^{-ik_x x_1(y,z)} - e^{-ik_x x_2(y,z)}) \quad (6)$$

that is equivalent to the system of two real-valued transcendent equations[13]. It is also possible to generalize this equation for targets with intrusions described by functions $x_3(y,z)$, $x_4(y,z)$ [16].

In one-dimensional problems, to retrieve the profile of an inhomogeneity $\varepsilon_1(z)$, equation (2) can be expressed as plane wave decomposition; however, its iterative solving may lead to poor results in cases of strong inhomogeneities [10], and it is better to represent it as the functional equation [17]:

$$\mathbf{E}_1^{\parallel,\perp}(\kappa_x,\kappa_y,z=0) = R^{\parallel,\perp}[\varepsilon_1](\kappa_x,\kappa_y)\mathbf{E}_0^{\parallel,\perp}(\kappa_x,\kappa_y,0). \quad (7)$$

Then, this inverse problem consists of analysis of reflection coefficients at TH and TE polarizations of plane waves or analysis of some functionals of these coefficients that can be calculated for any one-dimensional inhomogeneity. To retrieve the profile of permittivity inhomogeneities $\varepsilon_1(z)$ , it is possible to use the frequency dependence of the informative part of reflection coefficients of a plane wave $\Delta r[\varepsilon_1](\omega) = R[\varepsilon_1](\omega) - R[0](\omega)$. The solution is simplified, if the complex permittivity is determined by a single parameter of inhomogeneity, such as a dimensionless coefficient $f(z)$ [12,14,17] that determines permittivity in mixing formulas or conductivity $\sigma \approx \varepsilon\omega/4\pi i$ at low-frequency sounding of the Earth's crust. Then, the problem of the medium diagnostics is formulated as the condition $\Delta r[f](\omega) = \Delta r_0(\omega)$, where $\Delta r_0(\omega)$ denotes measured data. To solve this problem, a new method of dual regularization has been applied. At that, the modified Lagrange functional

$$L_\mu[f](\boldsymbol{\lambda}) = \|f\|^2 + \int_{\Delta\omega}\langle\boldsymbol{\lambda}(\omega), D[f](\omega)\rangle d\omega + \mu\{(\int_{\Delta\omega}|D[f](\omega)|^2 d\omega)^{1/2} + \int_{\Delta\omega}|D[f](\omega)|^2 d\omega\}, \quad (8)$$

where $D = \mathbf{\Delta r}[f](\omega) - \mathbf{\Delta r_0}(\omega)$, is minimized over $f$ at the simultaneous maximization of the dual functional

$$V_\mu^\alpha[f](\boldsymbol{\lambda}) = \min_{f\in D} L_\mu[f](\boldsymbol{\lambda}) - \alpha\|\boldsymbol{\lambda}\|^2 \to \max_{\|\lambda\|\le\mu}, \quad (13)$$

over Lagrange coefficients $\boldsymbol{\lambda}$ [5,17]. The saddle point of this process gives the desired solution. If the power reflection coefficient is measured [10,17], the phase information is lost, and the condition $\Delta r[\varepsilon_1](\omega) = |R[\varepsilon](\omega)|^2 - |R[0](\omega)|^2$ should be satisfied. To compensate this lost, it is possible to use *a priori* information about the set membership of the desired solution to one of compact function classes, such as monotonous or convex using decomposition of the solution over corresponding basic vectors $f = \sum a_j T^{(j)}$ . The proposed method can be applied in analysis of diffuse inhomogeneities in multilayer nanostructure of X-ray optics as well as for synthesis of aperiodic multilayer structures with desired reflection properties. In the latter case, the right side of (11) should be considered as an objective function [17].

## 3. LAST RESULTS

### 3.1 Metamaterial Lens in Tomography and Holography

Possibilities to apply non-perfect (absorbing) metamaterial lens to tomography and holography [11] in conditions of their mismatch to surrounding media have been studied and results are demonstrated in Figs. 1, 2. This mismatch leads to reflection on interfaces and multiple focusing in the sounded media.

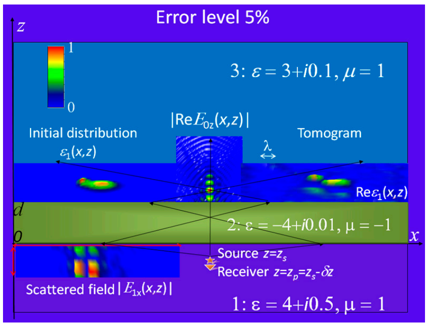


*Figure 1. Metamaterial lens tomography (vertical section). Layer 1: (left) received scattered field in the range of scanning (marked with red arrows); (center) source-receiver system. Layer 2: lens. Layer 3: (left) initial distribution of inhomogeneities; (center) z-component of the probing field amplitude; (right) retrieved distribution of inhomogeneities (tomogram).*

In Fig. 1, the numerical simulation of tomography for two gauss-distributed inhomogeneities is demonstrated. One can see that main details are retrieved quite right, but there are also some artefacts below true images. In Fig. 2, simulation results of holography is given for an example of solid targets – a parallelepiped with sizes 2.5 × 3.2 × 1.5 (in wavelength units).

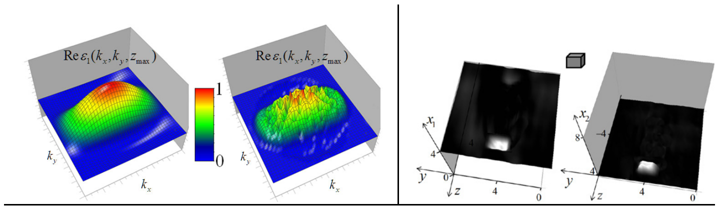

*Figure 2. Left, initial and retrieved (at 5% data error) transversal spectrum* $\mathrm{Re}\,\varepsilon(k_x,k_y)$ *at the depth of the target center; right, holography images represented by functions* $x_1(y, z)$, $x_2(y, z)$. *The target is shown in insertion.*

One can see that, unlike tomography results in Fig. 1, there are no visible artefacts in the holography images in Fig.2. Notwithstanding the absence of subwavelength resolution in tomography and holography with nonperfect lens, results demonstrate its applicability for subsurface diagnostics of media with unplanar, but soft surfaces, such as living tissues or soils.

### 3.2 Possible Applications of Multifrequency Microwave Near-Field Measurements in Medical Diagnostics

Possibilities to apply the microwave multifrequency tomography and holography [13] in applications to strongly absorbing media, such as living tissues, has been studied for the same source-receiver scanning system as in [13] that works in 1.7 – 7 GHz frequency band. In Fig. 3 (left), distributions of total and scattered fields are shown in *x-z* section for the tumor tissue 4×4×1 cm$^3$ in the fat medium; in Fig. 3 (right) – results of holography analysis by simulated data of the system [14] at the 5% rms level of random errors.

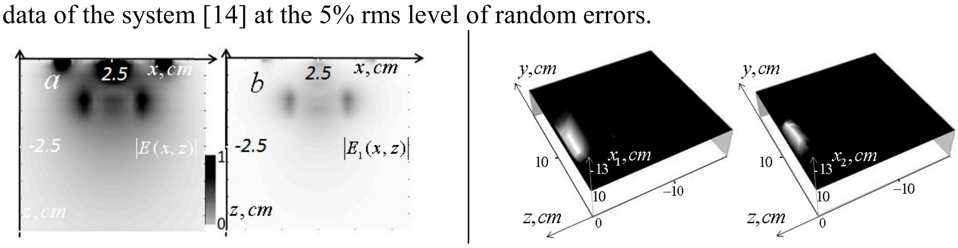


*Figure 3. Left, distributions of total and scattered electrical fields; right, results of holography analysis for the fat target, obtained from data of multifrequency measurements as functions* $x_1(y, z)$, $x_2(y, z)$.

Results in Fig. 3 demonstrate possibilities of microwave diagnostics of targets in rather low-absorbing living tissues, such as fat; however in the muscle tissues it is possible only up to depths ~2 cm. The scattered field is small enough in this case that provides a good applicability of the Born approximation. For deeper sounding it is necessary to use a lower frequency band, for example, *f* = 0.6 – 2 GHz.

### 3.3 Simulation of Low-Frequency Diagnostics of Targets with Intrusions

Possibilities of subsurface multifrequency electromagnetic holography of targets with intrusions [16] have been studied for the case of low-frequency diagnostics of Earth's crust conductivity inhomogeneities [4] by data of multifrequency measurements of complex amplitudes of the magnetic field component $H_{1y}$ ($z = 0$) in the range 0.25 – 2500 Hz transformed into the synthesized pseudopulse as a function of the parameter $z_s$.

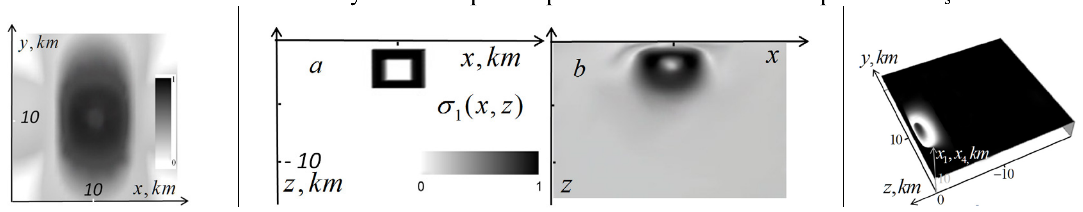


*Figure 4. Left, transversal distribution of pseudopulse* $|H_{1y}(x,y,z_s{=}-2\ km)|/H_{max}$*; center, target with an intrusion and its tomogram in x-z section; right, hologram image of the target (represented by function* $x_1(y,z)$*) with intrusion represented by function* $x_4(y,z)$.

Results of numerical simulation in Fig. 4 demonstrate the possibility to retrieve such complicated inhomogeneities at the 3% error level; however, for targets below $z = -3$ km it requires such a high level of the measurement accuracy that hardly can be realized in practice.

In conclusion, it is also worth mentioning the possibility to apply these methods in acoustic diagnostics [18].

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