Pulse and Multifrequency Near-Field Subsurface Diagnostics

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ABSTRACT
In this paper, last results of theoretical and experimental studies of the near-field non-invasive diagnostics of subsurface dielectric inhomogeneities are presented. New methods of this diagnostics based on the sounding with pulse signals is proposed and demonstrated. Like in the multifrequency method, measurements of the scattered signal are in use to solve corresponding inverse scattering problems. Solving algorithms for sounding of various kinds of inhomogeneities: profiling of one-dimensional inhomogeneities, tomography of 3D distributed inhomogeneities, or computer holography of the shape of solid subsurface targets are proposed. The study includes the comparison of the pulse method with the early-developed multifrequency diagnostics.
Keywords: electromagnetic inverse scattering problems, tomography, holography.

1. INTRODUCTION

The near-field diagnostics is based on the solution of the inverse scattering problem by parameters of signals measured above the medium surface scattered by dielectric subsurface targets that are placed in the near zone. In this case, it appears possible to overcome the Rayleigh limitation of the achievable resolution that can be the only way when it is impossible to use shorter wavelengths because the growth of absorption in sounded media.

In papers [1-5], various approaches have been developed base on multifrequency measurements of scattered harmonic signals. In one-dimensional problems, to retrieve profile inhomogeneities of complex permittivity $\varepsilon_1(z)$, methods have been worked out based on the dual regularization theory – a new Lagrange approach in the theory of nonlinear ill-posed problems [5].

To retrieve 3D distributions of subsurface inhomogeneities of complex permittivity (i.e., to solve the problem of computer electromagnetic tomography), data of multifrequency, multilevel or multi-angle measurements obtained in 2D range above the buried targets were involved in analysis in various applications [1-4]. Methods of 3D diagnostics are based on the non-linear integral equation that is solved beginning with the Born approximation. For some measurement schemes, it is expressed as a convolution over transversal co-ordinates and, hence, can be reduced to one-dimensional integral equation. This equation should be solved multiply for all pairs of transversal spectrum component that makes it possible to overcome serious grid-size limitations in solving 3D problems. In particular, this approach can be applied to data of 2D scanning above the region with inhomogeneities at the fixed source-receiver relative positions. This method has been applied in the near-field tomography of 3D distribution of complex permittivity by measurements of signal complex amplitudes at 801 frequencies in the range 1.7 – 7.0 GHz with the source-receiver system composed of Agilent E5071B vector network analyzer and two identical transmitting and receiving planar bow-tie antennas in the bistatic configuration that were scanning together in the rectangle *x-y* area above the targets [3,4]. For solid targets with a homogeneous internal structure, it appeared possible to obtain their shape (i.e. to solve the problem of computer holography) with a subwavelength resolution [3,4].

In this paper, a new approach that is based on the pulse sounding is proposed to the near-field diagnostics.

2. TEORY

The proposed pulse diagnostics is based on measurements of scattered signal (pulse) $s(\mathbf{r}_r, t)$, where co-ordinates x_r and y_r of the vector $\mathbf{r}_r$ mark the transversal position of the receiver and z_r – its altitude above the sounded medium; t is time. This signal is determined by the distribution of the scattered field. In multifrequency diagnostics of 3D subsurface inhomogeneities, distributions of probing and scattered field of a harmonic signal can be calculated using Green function formalism [2]. However, for an arbitrary time-dependent probing field these Green functions can't be found. Because of this reason, it is suitable to use in analysis the Fourier decomposition of the pulse signal in its time interval Δt:

$$s(\mathbf{r}_r, \omega) = \frac{1}{2\pi}\int_{t_0}^{t_0+\Delta t} s(\mathbf{r}_r, t)\exp(-i\omega t)dt\,, \qquad (1)$$

where the initial time t_0 that determines the phase of complex amplitudes of spectral components. It is reasonable to choose it as the beginning of the returned pulse from the surface. This frequency spectrum can be expressed as a convolution of the field spectrum with the transfer function over lateral co-ordinates:

$$s(\mathbf{r}_r, \omega) = \int \mathbf{E}(\mathbf{r}', \omega)\mathbf{F}(x_r - x', y_r - y', z_r, z', \omega)dx'dy'dz'\,. \qquad (2)$$

Then, by 2D Fourier transform of (2) over transversal co-ordinates x_r and y_r the transversal spectrum of the received signal can be obtained [2-4]. For plane apertures of source and receiver that will be studied in numerical and real experiments $\mathbf{F}(x_r - x', y_r - y', z_r, z', \omega) = \mathbf{F}(x_r - x', y_r - y', \omega)\delta(z_r - z')$, and one has

$$s(k_x, k_y, \omega) = 4\pi^2 \mathbf{E}(k_x, k_y, \omega)\mathbf{F}(k_x, k_y, \omega)\,. \tag{3}$$

To simplify notation, above and below we mark Fourier components of frequency and space spectra by their arguments.

For a medium with complex permittivity ε_0 with a subsurface 3D inhomogeneity $\varepsilon_1 = \varepsilon_1' + i\varepsilon_1''$, so that $\varepsilon(\mathbf{r}) = \varepsilon_0 + \varepsilon_1(\mathbf{r})$, complex amplitudes of the electric field and received signal at frequency ω can be expressed as sums of probing and scattered components ($\mathbf{E} = \mathbf{E_0} + \mathbf{E}_1$; $s = s_0 + s_1$). For the above-mentioned scheme of measurements with the fixed source-receiver vector $\boldsymbol{\delta}\mathbf{r}$, the transversal spectrum of scattered field (2D Fourier transform over x and y) can be expressed in the Born approximation as [2]:

$$\begin{aligned} E_{1i}(k_x, k_y, \omega, \boldsymbol{\delta}\mathbf{r}) = -4\pi^3 i\omega \int_{z'} \varepsilon_1(k_x, k_y, z')\{\iint \mathrm{e}^{-i\kappa_x \delta x - i\kappa_y \delta y} \\ \times [j_i(\kappa_x, \kappa_y, z_r + \delta z, \omega) G_{ij}^{12}(\kappa_x, \kappa_y, z_r + \delta z, z', \omega)] G_{ji}^{21}(\kappa_x + k_x, \kappa_y + k_y, z', z_r, \omega) d\kappa_x d\kappa_y\} dz'. \end{aligned} \tag{4}$$

From expressions (3) and (4) obtain the integral equation for the scattered signal:

$$s_1(k_x, k_y, \omega) = \int_{z'} \varepsilon_1(k_x, k_y, z') K(k_x, k_y, z', \omega) dz, \tag{5}$$

$$\begin{aligned} K(k_x, k_y, z', \omega) = -4\pi^3 i\omega F_i(k_x, k_y, \omega) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-i\kappa_x \delta x - i\kappa_y \delta y} \\ \times [j_i(\kappa_x, \kappa_y, z_r + \delta z, \omega) G_{ij}^{12}(\kappa_x, \kappa_y, z_r + \delta z, z', \omega)] G_{ji}^{21}(\kappa_x + k_x, \kappa_y + k_y, z', z_r, \omega) d\kappa_x d\kappa_y, \end{aligned} \tag{6}$$

where the kernel K is expressed by components of transversal spectra of current $\mathbf{j}$ and Green functions G_{ij}^{12}, G_{ij}^{21} for waves propagating from medium 1 to medium 2 (sounded medium) and in the backward direction. Explicit formulas for Green functions can be found in [2].

The integral equation (6) coincides formally with the equation that has been obtained in [3,4] for the multifrequency diagnostics, however, in this case data are obtained from the Fourier decomposition of the pulse signal. In this case, the frequency step and the depth of the frequency band $\Delta\omega$ of this transform become free parameters. Also, taking into account that the signal is a real value, it is enough to use only positive part of the signal spectrum in analysis. The equation (6) should be solved for each pair of components k_x, k_y. In solving this Fredholm integral equation of the 1-st kind, the algorithm of Tikhonov's method of generalized discrepancy developed in [2] for complex-valued functions can be applied. As a result, from the solution of the problem in k-space one has the solution of the tomography problem – the desired 3D distribution of complex permittivity of a sounded inhomogeneity:

$$\varepsilon_1(x, y, z) = \iint \varepsilon_1(k_x, k_y, z) \exp(ik_x x + ik_y y) dk_x dk_y. \tag{7}$$

Also, in the same way as in the multifrequency diagnostics [3,4], it is possible to propose the method of computer holography – the retrieval of the outer shape of solid targets with a homogeneous internal structure that are very typical in practice. If it is known *a priori* that the target permittivity ε_1^0 = const, this problem can be solved using the solution $\varepsilon_1(\kappa_x, \kappa_y, z)$ of (6) in k-space as two functions $y_1(x,z),\ y_2(x,z)$ from the solution of the transcendent complex-valued equation that is equivalent to the system of two real-valued equations:

$$\begin{aligned} \varepsilon_1(k_y, x, z) = \frac{\varepsilon_1^0}{2\pi i k_y}(e^{-ik_y y_1(x,z)} - e^{-ik_y y_2(x,z)}). \\ \varepsilon_1(k_y, y, z) = \int_{-\infty}^{\infty} \varepsilon_1(k_x, k_y, z) \exp(ik_x y) dk_x, \end{aligned} \tag{8}$$

These two functions determine the shape of a target. It is worth mentioning that the system (6) is over determined that makes it possible to optimize algorithms.

In the case of one-dimensional inhomogeneities (subsurface permittivity profiles, the scattered field is determined by the incident field and the reflection coefficients on TH and TE polarizations [5]:

$$\mathbf{E}_1(\kappa_x, \kappa_y, z_r) = [R^{\parallel}(\kappa_x, \kappa_y)\mathbf{E}_0^{\parallel}(\kappa_x, \kappa_y, 0) + R^{\perp}(\kappa_x, \kappa_y)\mathbf{E}_0^{\perp}(\kappa_x, \kappa_y, 0)] \exp(\sqrt{k^2 - \kappa_x^2 - \kappa_y^2}\, z_r)\,. \tag{9}$$

$$\mathbf{E_0}(\kappa_x,\kappa_y,0) = -\frac{2\pi}{\omega}\exp(i\sqrt{k^2-\kappa_x^2-\kappa_y^2}z_s)\Bigg\{ j_x(\kappa_x,\kappa_y,z_s)\{\left[\frac{\kappa_x^2 k_z}{\kappa_\perp^2}\vec{x}_0 + \frac{\kappa_x\kappa_y k_z}{\kappa_\perp^2}\vec{y}_0 - \kappa_x\vec{z}_0\right]_{\parallel} + \frac{k^2}{\kappa_\perp^2 k_z}\left[\kappa_y^2\vec{x}_0 - \kappa_x\kappa_y\vec{y}_0\right]_\perp\}$$

$$+ j_y(\kappa_x,\kappa_y,z_s)\{\left[\frac{\kappa_x\kappa_y k_z}{\kappa_\perp^2}\vec{x}_0 + \frac{\kappa_y^2 k_z}{\kappa_\perp^2}\vec{y}_0 - \kappa_y\vec{z}_0\right]_{\parallel} + \frac{k^2}{\kappa_\perp^2 k_z}\left[-\kappa_x\kappa_y\vec{x}_0 + \kappa_x^2\vec{y}_0\right]_\perp\}\Bigg\},$$

where $k = \omega / c$, $k_z = \sqrt{k^2-\kappa_x^2-\kappa_y^2}$, $\kappa_\perp = \kappa_x^2+\kappa_y^2$, $z_s = z_r + \delta z$ is the source altitude above the surface.

There could be a very typical case when reflection coefficients in (9) are determined by the frequency-dependent complex permittivity $\varepsilon(z,\omega)$, and we deal with an insolvable 2D problem. In this case, it is necessary to find a proper one-dimensional parameter that determines permittivity, such as the profile of volume water content $f(z)$ [5] or the material density $f(z)$. For identical source and receiver apertures, it is possible to use the reciprocity condition $F_i(\kappa_x,\kappa_y,\omega) = const\ j_i(\kappa_x,\kappa_y,\omega)$ at the calculation of the received signal in (3). Then the problem can be formulated like this: to find the profile $f(z)$ that satisfy the equation between calculated and measured data:

$$s[f(z)](z_s)) = s_0(z_s). \tag{10}$$

Because the direct problem is solved much easily in this one-dimensional problem, it is possible to use in solving more powerful approach – the method of dual regularization [5]. To calculate the signal, it is enough to calculate reflection coefficients for plane waves reflected from the half-space with the one-dimensional profile of permittivity. So, according to [5], the modified Lagrange function in the dual regularization method can be written as

$$L_\mu[f](\boldsymbol{\lambda}) = \|f_w\|^2 + \frac{1}{\Delta\omega}\int_\omega \langle \boldsymbol{\lambda}(\omega),(\mathbf{s}[f](\omega)-\mathbf{s_0}(\omega))\rangle d\omega$$
$$+\mu\{\left(\frac{1}{\Delta\omega}\int_\omega |\mathbf{s}[f_w](\omega)-\mathbf{s_0}(\omega)|^2 d\omega\right)^{1/2} + \frac{1}{\Delta\omega}\int_\omega |\mathbf{s}[f](\omega)-\mathbf{s_0}(\omega)|^2 d\omega\}, \tag{11}$$

where $\langle\cdot\rangle$ is the scalar product, $\|f\|^2_{L_2} = \frac{1}{\Delta z}\int_{\Delta z} f(z)^2 dz$, $\boldsymbol{\lambda} = (\lambda_1,\lambda_2)$, $\mu > 0$. The regularized dual problem is a problem of maximization of the concave functional

$$W_\mu(\boldsymbol{\lambda}) = \min_{\sigma\in D} L_\mu[f_w](\boldsymbol{\lambda}) - \alpha\|\boldsymbol{\lambda}\|^2 \to \max_{\|\lambda\|\le\mu}, \tag{12}$$

where $D = \{f \in L_2(z_n,0): 0 \le f(z) \le f_{\max}\}$. Here the complex-valued reflection signal **s** is considered as a two-dimensional vector. The supergradient of the functional (12) is expressed explicitly. The desired solution is obtained as a saddle point of the process of minimization of (11) over f at the maximization of (12) over the dual variable $\boldsymbol{\lambda}$.

This problem should be solved for data with a finite error, so that

$$\|\mathbf{s}[f(z)] - \mathbf{s}_0\|^2_{L_2} \equiv \frac{1}{\Delta\omega}\int_\omega |\mathbf{s}[f(z)](\omega) - \mathbf{s}_0(\omega)|^2 d\omega \le \delta_R^2,$$

where $\Delta\omega$ determines the region of data used in analysis. At that, we assume $\mu = 10$ in (41), and begin the iterative procedure with $\boldsymbol{\lambda}^{k=1} = 0$, $\partial W_\mu^{\alpha(k=1)}(\boldsymbol{\lambda}) = \frac{1}{\Delta z_s}\{\mathbf{s}[f^{k=1}(z) = \sigma_0](\omega) - \mathbf{s}_0(\omega)\} - 2\alpha\boldsymbol{\lambda}$, and calculate the initial discrepancy $\delta^{k=1} = \left\|\mathbf{s}[f^{k=1}(z) = f(z=0)] - \mathbf{s}_0\right\|_{L_2}$. In the iterative procedure (18) further values of discrepancy $\delta^k = \left\|\mathbf{s}[f^k(z)] - \mathbf{s}_0\right\|_{L_2}$ are calculated; at that, we used sequences $\alpha^k = k^{-1/3}$, $\beta^k = 10^{-2}k^{-1/2}$, $k = 1,2...$ that satisfy necessary conditions in the theory. Based on heuristic reasons and numerical simulations, stopping rules for developed algorithms have been proposed in [5]. The iterative procedure proceeds up to the largest number $k = k(\delta_R)$, for which one of stopping rules $\left\|\partial V_\mu^{\delta_R}(\boldsymbol{\lambda}^k)\right\| > a$, $\delta^k \ge b\delta_R$ is fulfilled (*a, b* are values determined from the numerical simulation). The last of these two rules is, in fact the discrepancy condition. The corresponding point f^k gives us the desired solution of the problem.

3. NUMERICAL SIMULATION

These methods of the proposed near-field pulse diagnostics are tested for the same microwave scanning system that has been applied in the multifrequency diagnostics [3,4] that includes two identical transmitting and receiving planar bow-tie antennas in the bistatic configuration. They were scanning together in the rectangle *x*-*y* area above the targets.

In Fig. 1 one can see the probing pulse and results of numerical simulation of one-dimensional pulse diagnostics of a gauss-form inhomogeneity of the sand density by measurements of the scattered pulse. Results have been obtained with the dual regularization method (10)-(11).

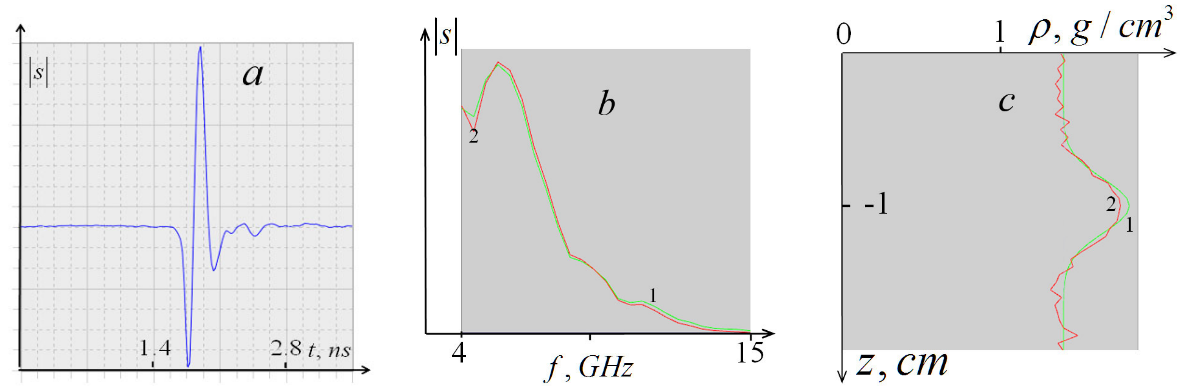

Figure 1: a, probing pulse; b, 1 – spectrum of scattered signal from the inhomogeneity of the sand density (line 1 in Fig. 1c), 2 – spectrum of retrieved profile of density (line 2 in Fig. 1c); c, 1 – simulated inhomogeneity of sand density, 2 – retrieved profile. Solution is obtained at 5% level of random data errors; $f = \omega / 2\pi$.

The sharp probing pulse (Fig.1a) produces a broad spectrum that gives a rather broad spectrum of the scattered signal (Fig. 1b).

In Fig. 2, results of the holography simulation based on the solution of (8) are demonstrated for the simulated foam target with sizes 3×3×1 cm^3 buried in the sand at the depth z_t = -7 cm.

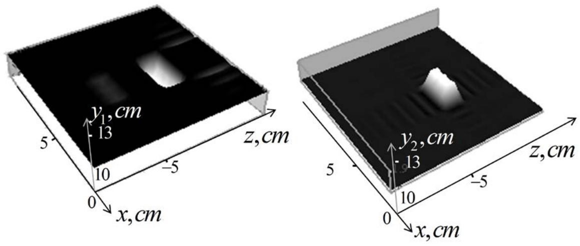

Figure 2. Left, holography image of the target described by function $y_1(x,z)$ *; right, image of the target described by function* $y_1(x,z)$.

Results of this simulation demonstrate feasibility of proposed methods of near-field pulse diagnostics. Their application to experimental data will be presented.

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