Metamaterial Lens Tomography

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ABSTRACT
Possibilities of the coherent scanning tomography with negative-index metamaterial lens are studied. Such lens (a slab of negative refractive index material) has the power to focus the source field into a sounded media with dielectric inhomogeneities. By the 2D scanning with a source-receiver system at a number of levels from the lens surface, this focus can be immersed far into the studied region that makes it possible to realize the proposed tomography. The solution of the corresponding inverse scattering problem, i.e., the retrieval of 3D distributions of the complex permittivity in the inhomogeneous region, is based on the solution of the 3D integral equation for the scattered field that is reduced to the one-dimensional Fredholm integral equation of the 1-st kind relative to the depth profile of lateral spectrum of inhomogeneities. After the solution of this equation for each pair of spectral components, the inverse Fourier transform leads to the desired solution. The feasibility of this tomography for low-contrast targets is demonstrated in numerical simulation.
Keywords: inverse problem of scattering, scanning tomography, negative-index metamaterials.

1. INTRODUCTION

As it was shown in [1], a slab of negative refractive index material ($\varepsilon = -1$, $\mu = -1$) has a property to focus all the Fourier components of a source, that leads to the perfect resolution of the 2D image of objects in the focal plane. This focus can penetrate far into the studied region that opens a new possibility of the subwavelength scanning tomography. Propagating and evanescent wave components meet in the outer focus at $z_f = 2d - z_s$ (as if the source at $z = z_s$ were transferred to this point (see in Fig. 2). Varying the source vertical position, this focus can be moved into the studied region inside the interval $d < z_f < 2d$. Hence, the exact image of an object in the plane $z_f = 2d$ can be formed by this lens [1]. It seemed possible to obtain the exact tomography of a 3D object by 2D lateral scanning at several levels behind the lens with the source-receiver system, changing the position of the focus throughout the studied region. The general scheme of multilevel scanning tomography, developed in [2-3], has been applied to this inverse problem and studied in numerical simulation in [4-6], where the subwavelength resolution has been demonstrated.

However, even a very small absorption in left-handed materials restricts drastically the range of subwavelength focusing [7]. Nevertheless, the possibility to use the property of plane lenses to focus the probing field into a studied region remains very attractive for the subsurface tomography. Such a tomography is studied here for left-handed metamaterial lenses with various parameters. The scheme of the proposed tomography, like in [2-6], is based on the method of data acquisition at the condition of the fixed emitter-receiver distance, invented in [3]. At this condition, the 2D Fourier transform (plane wave decomposition) reduces the 3D integral equation for the scattered field to the one-dimensional Fredholm integral equation of the 1-st kind relative to the depth profile of lateral spectrum of permittivity inhomogeneities. Beginning with the Born approximation, this method can be generalized beyond this approximation [6]. The feasibility of such tomography methods is shown here in numerical modeling.

2. INVERSE PROBLEM OF SCATTERING

Let us consider the inverse problem of scattering in the proposed scheme of the metamaterial lens tomography shown in Fig.2. The source and receiver are placed in the layer 1; the layer 2 is the metamaterial lens; the studied 3D inhomogeneous region $\varepsilon_1(\boldsymbol{r})$ is imbedded in the layer 3. The reference (unperturbed) field $\mathbf{E}_0(\mathbf{r})$ that is used in the sounding is determined by the proper Green tensor (that includes near-field components):

$$\mathbf{E}_0(\mathbf{r}) = \int_V \mathbf{j}(\mathbf{r}')\vec{\mathbf{G}}(x-x', y-y', z, z')d\mathbf{r}' , \tag{1}$$

where $\mathbf{j}(\mathbf{r})$ is the current distribution in the source. Using the plane wave decomposition of (1) over lateral co-ordinates, it is possible to obtain $\mathbf{E}_0(\mathbf{r})$ in any layered medium [6]. In the presence of the scattering region, the electric field is determined as the sum of the reference and the scattered fields $\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \mathbf{E}_1(\mathbf{r})$ by known Neumann series. The solution of the inverse problem requires the solution of the corresponding 3D non-linear integral equation that can be solved iteratively, beginning with the Born approximation [3, 6]. It is a very complicated problem, but a method is proposed in [3] to reduce this equation to a convolution equation over lateral co-ordinates. For this, it is enough to fix the source-receiver spacing $\boldsymbol{\delta}\mathbf{r}$ (it could be done as well at the stage of data processing). At this condition, the sounding field structure will be invariable relative to the receiver position that leads to the significant improvement of the accuracy of measurements because all observed variations are related only to studied inhomogeneities. This representation makes it possible to reduce the 3D integral equation to 1D integral equation relative to the depth profile of the lateral spectrum of ε_1, using 2D Fourier transform over transverse co-ordinates [3, 6]:

$$E_{1i}(k_x,k_y,z) = -4\pi^3 i\omega \int_{z'} \varepsilon_1(k_x,k_y,z')[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-i\kappa_x\delta x - i\kappa_y\delta y}$$
$$\times \int_{z''} [j_i(\kappa_x,\kappa_y,z''-z-\delta z) G_{ij}^{13}((\kappa_x,\kappa_y,z'',z')] G_{ji}^{31}(\kappa_x+k_x,\kappa_y+k_y,z',z) d\kappa_x d\kappa_y dz'' dz', \quad (2)$$

$$G_{ji}^{13}(k_x,k_y,z'',z') = -\frac{1}{2\pi\omega}\frac{k_0^2}{k_\perp}\exp i\left\{-\sqrt{k_1^2-k_\perp^2}z'' + \sqrt{k_3^2-k_\perp^2}(z'-d) - \sqrt{k_2^2-k_\perp^2}d\right\}$$
$$\times[\frac{1}{k_1k_3}T_{13}^{\parallel}\begin{Bmatrix} \frac{k_x^2k_{z3}}{k_\perp} & \frac{k_xk_yk_{z3}}{k_\perp} & -k_xk_\perp \\ \frac{k_xk_yk_{z3}}{k_\perp} & \frac{k_y^2k_{z3}}{k_\perp} & -k_yk_\perp \\ -\frac{k_xk_\perp k_{z3}}{k_{z1}} & -\frac{k_xk_\perp k_{z3}}{k_{z1}} & \frac{k_\perp^3}{k_{z1}} \end{Bmatrix} + \frac{1}{k_{z1}k_\perp}T_{13}^{\perp}\begin{Bmatrix} k_y^2 & -k_xk_y & 0 \\ -k_xk_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{Bmatrix}],$$

$$G_{ji}^{31}(k_x,k_y,z',z) = -\frac{1}{2\pi\omega}\frac{k_0^2}{k_\perp}\exp i\left\{-\sqrt{k_1^2-k_\perp^2}z + \sqrt{k_3^2-k_\perp^2}(z'-d) - \sqrt{k_2^2-k_\perp^2}d\right\}$$
$$\times[\frac{1}{k_1k_3}T_{31}^{\parallel}\begin{Bmatrix} \frac{k_x^2k_{z1}}{k_\perp} & \frac{k_xk_yk_{z1}}{k_\perp} & k_xk_\perp \\ \frac{k_xk_yk_{z1}}{k_\perp} & \frac{k_y^2k_{z1}}{k_\perp} & k_yk_\perp \\ \frac{k_xk_\perp k_{z1}}{k_{z3}} & \frac{k_yk_\perp k_{z1}}{k_{z3}} & \frac{k_\perp^3}{k_{z3}} \end{Bmatrix} + \frac{1}{k_{z3}k_\perp}T_{31}^{\perp}\begin{Bmatrix} k_y^2 & -k_xk_y & 0 \\ -k_xk_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{Bmatrix}].$$

where $k_0 = \omega / c$, $k_\perp^2 = k_x^2 + k_y^2$, G_{ji}^{lk} are components the lateral 2D Fourier transform of the Green function **G** that determine the contribution of j^{th} component of the source to i^{th} component of the field at the receiver position in the k^{th} layer and the position of the sounded region in the l^{th} layer. These functions and transfer coefficients $T_{ij}^{\parallel,\perp}$ for TM and TE polarizations can be obtained for arbitrary multilayer media [6]. In (2) the concrete form of Green tensors in different layers is given, taking into account the wavevector transformation.

So, we have to solve the one-dimensional Fredholm integral equation of the 1-st kind relative to the vertical profile of permittivity for each pair of lateral spectral components. The kernel of this equation depends on the vertical position of the receiver z that determines the focus position in the sounded medium, and this dependence determines the spatial resolution of tomography.

To retrieve complex-value profiles $\varepsilon_1(k_x,k_y,z')$ from the one-dimensional Fredholm integral equation of the 1-st kind (2), the mathematically consistent algorithm based on the Tikhonov's principal of generalized discrepancy has been developed for complex-value functions in the W_2^1 Hilbert space (Sobolev's space) [5, 6]. The regularization parameter is determined by the integral error of lateral spectrum of the scattered field. This spectrum error can be derived from the known integral error of the measured signal using the Plansherel's theorem. Finally, the desired 3D structure of permittivity is obtained by 2D inverse Fourier transform of the retrieved profiles of lateral spectrum components. It is also possible to take into account the transfer function of the receiver in the same way as in [8], where the integral equation (2) has been used in method of multifrequency subsurface tomography in the microwave range.

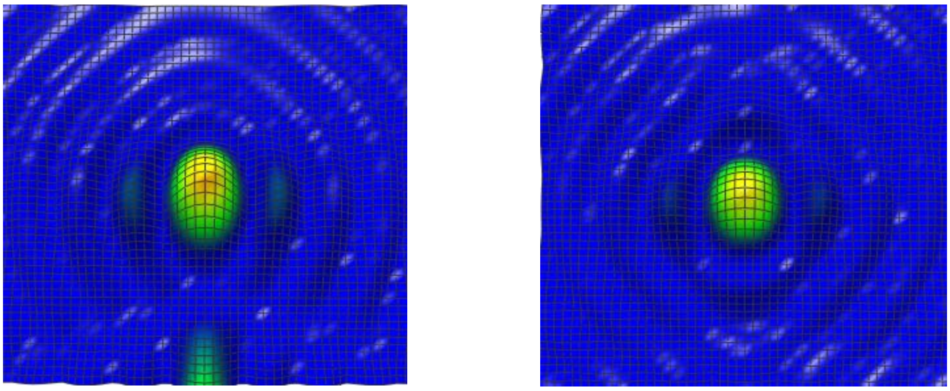

Figure 1. The field distribution above the metamaterial lenses with different level of absorption. Left: $\varepsilon = -1 + i0.001$, $\mu = -1$; right: $\varepsilon = -1 + i0.01$, $\mu = -1$.

In Fig. 1 distributions of probing field (vertical section $E_{0z}(x,z)$ for the point source used in the numerical simulation of tomography shown in Fig. 2) are given at two different level of metamaterial absorption. Only a weak field enhancement near the lens surface is seen on the left image (at the very small absorption $\varepsilon'' = 0.001$ in the metamaterial lens) instead of the surface-focus field singularity behind a perfect lens (without absorption). This residual enhancement is related to the focusing of near-field components. One can see that at a small enough increase of absorption up to $\varepsilon'' = 0.01$ (on the right image in Fig. 1) this residual near-field focusing disappears, and field distribution is determined mainly by propagating field components.

3. NUMERICAL SYMULATION

Results of the numerical simulation of the proposed multilevel scanning tomography are shown in Fig. 2.

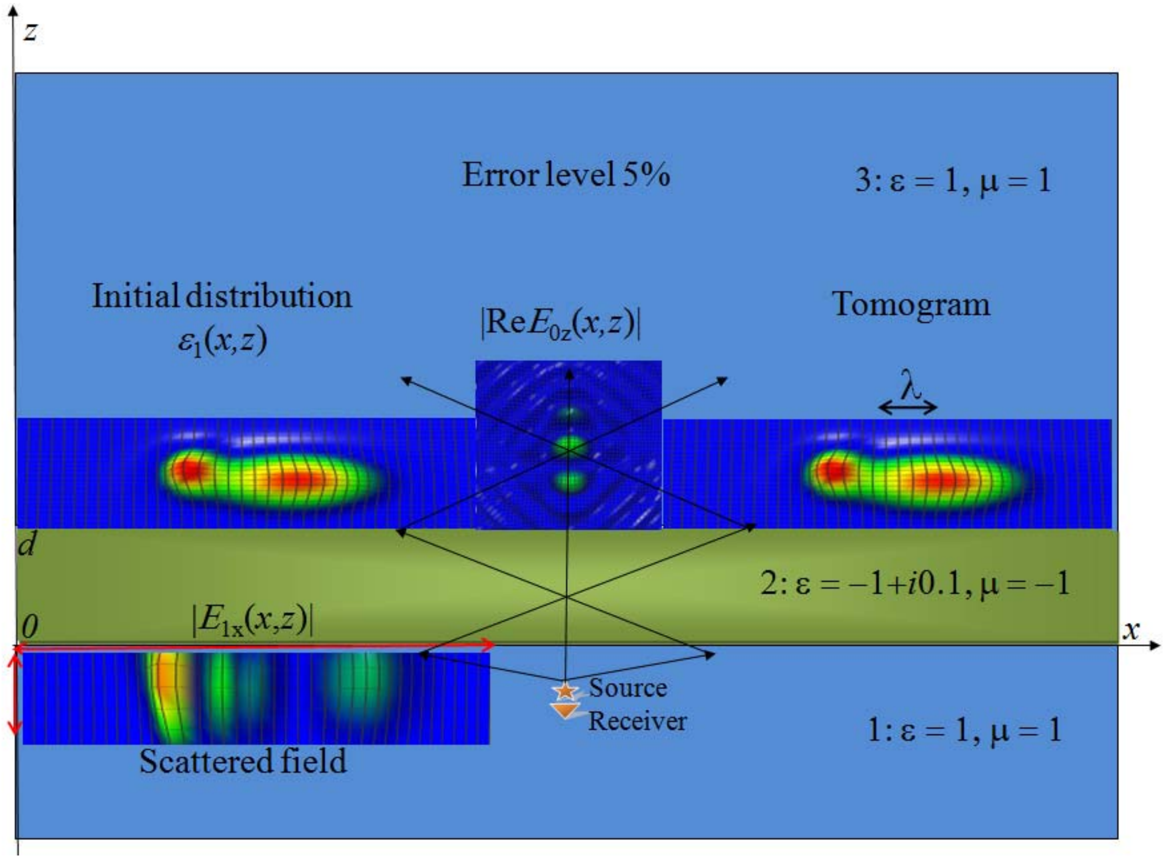

Figure 2. Metamaterial lens tomography (vertical section). Layer 1: (left) received scattered field in the range of scanning (marked with red arrows); (center) source-receiver system. Layer 2: lens. Layer 3: (left) initial distribution of inhomogeneities; (center) z-component of the probing field; (right) retrieved distribution of inhomogeneities (tomogram).

To retrieve the simulated distribution of 3D complex permittivity $\varepsilon(\boldsymbol{r})$, the *x*-component of the scattered field $E_{1x}(x, y, z_i + \delta z)$ "measured" at the vertical distance $\delta z = -0.1\lambda$ from *z*-component point source of the probing field $\mathbf{j}(\mathbf{r}''-\mathbf{r}-\delta\mathbf{r}) = j_z\delta(\mathbf{r}''-\mathbf{r}-\delta z)\mathbf{z_0}$ at 6 levels $z = z_i$ in *x-y* plane below the lens (in the range marked with red arrows in Fig.2) has been used in analysis. The simulated inhomogeneous region above the metamaterial lens was chosen as the sum of two gauss asymmetry distributions. Their sizes are larger than wavelength λ (marked with black arrow). Results of the tomography at 5% level of random errors, shown in Fig.2 for the real part of simulated inhomogeneous permittivity, demonstrate a high-quality retrieval of the initial distribution. The quality of retrieval for real and imaginary parts of permittivity in the simulated target is similar.

The study of the proposed tomography has been carried out for various media parameters of layers in the scheme shown in Fig. 2. It was obtained that results are much better, if layers' parameters are matched (by absolute values). If they are not matched, at least, they shouldn't be defocusing. The focusing can be also destroyed by the absorption in the lens material. It is possible to see this effect (the focus blurring in the vertical direction), comparing the probing field distribution at $\varepsilon'' = 0.01$ (Fig. 1, right) and at $\varepsilon'' = 0.1$ (Fig. 2).

This method could be useful in subsurface electromagnetic diagnostics of inhomogeneities in various media using probing field in properly chosen spectral ranges. The application of plane lens could be very suitable for diagnostics of soft media with rough or uneven surfaces, such as human skin, for example, to avoid the influence of this roughness on the signal formation.

It is also worth mentioning, that unlike the tomography with perfect (non-absorbing) tomography, in the considered here tomography with non-perfect lenses it is possible to use not only the multilevel scheme, but also multifrequency measurements with a single 2D scanning along the lens surface. In this case, the source-receiver-lens system can be manufactured as a single whole. The corresponding multifrequency inverse scattering problem has been considered in the numerical simulation [6] and, then, applied in the multifrequency microwave tomography of underground dielectric targets in [8]. Such multifrequency measurements couldn't be applied for tomography with the use of perfect lens by reason of extremely resonant properties of ideal left-handed metamaterials. For considered here non-perfect lens, resonant properties of metamaterials can be effectively used to change the depth of the focus position. It makes multifrequency measurements depth-sensitive and the multifrequency tomography – feasible.

4. CONCLUSIONS

The feasibility of the proposed method of scanning tomography with the use of the left-handed metamaterial lenses has been demonstrated in numerical simulation.

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