

# IGARSS'94

*International Geoscience and Remote Sensing Symposium*

*Surface and Atmospheric Remote Sensing:  
Technologies, Data Analysis and Interpretation*

*Volume IV*

*California Institute of Technology*

*Pasadena, California USA*

*August 8-12, 1994*



*IEEE Catalog Number 94CH3378-7 Library  
of Congress Number: 93-80348*

**1994 International Geoscience and Remote Sensing Symposium**

Editor: Tammy I. Stein

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IEEE Catalog Number: 94CH3378-7  
ISBN (softbound edition): 0-7803-  
1497-2 ISBN (casebound edition): 0-  
7803-1498-0 ISBN (microfiche  
edition): 0-7803-1499-9 Library of  
Congress Number: 93-80348

# Tikhonov's Method of the Ground-Based Radiometric Retrieval of the Ozone Profile

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## Abstract

The incorrect problem of the ozone height profile retrieval by ground-based measurements of the brightness temperatures in ozone spectral lines has been considered elsewhere only by using recurrence numerical methods based on properly chosen height discretization. Such approach is mathematically inconsistent and does not allow to clear up the true retrieval possibilities.

The method based on the Tikhonov's theory has been worked out to solve the problem and numerical simulation in the lines centered at 110.8 and 142.2 GHz is presented. The influence of measurement errors, spectral band width, and channel number in the spectral band on retrieval of the ozone profile peculiarities at different height levels is investigated.

## INVERSE PROBLEM FORMULATION

### Introduction

One of the most important remote sensing problems appears now a problem of atmosphere ozone concentration monitoring which can make possible to investigate the influence of various factors on ozone height distribution. Method of height profile retrieval uses radiometric measurements in spectral lines of O<sub>3</sub> in millimeter range (resonance frequencies in use: 101, 110, 142, 184 GHz).

The space-borne as well as ground-based methods have been worked out. From space-borne methods the limb-viewing method is the most sensitive. In that case the ozone profile is retrieved from brightness temperature dependence on ray tangent height and the problem consist in the solution of integral Volterra equation of the 1-st kind with the variable lower limit. For it's solution both statistical (Waters, 1981) and non-statistical (Gaikovich, 1991) methods have been proposed. The preferences of limb-viewing method are stability of the solution with respect to measurements errors and the height vertical resolution. But there is a difficulty - the horizontal homogeneity in the range 500 - 1000 km is supposed and the horizontal resolution is also 500 - 1000 km.

And vice versa, in ground-based inverse problem one has the exact horizontal position of the profile but the vertical resolution is much worse. The correspondent inverse problem is considered elsewhere (see, for example, Randegger, 1980, Monnantenil, 1983, Lobsinger, 1984, Solomonov, 1990, Kulikov, 1988, Brillet J., 1989). The height profile of ozone is retrieved by brightness temperature dependence on frequency in one of it's

spectral lines. The problem consists of the solution of integral Fredholm equation of the 1-st kind. It is well known that the solution of that incorrect problem is impossible without using of additional a priori information on the exact solution. The solution precision depends substantially on the a priori information specifics as well as on the class of functions to which belongs the retrieved profile. The convergence of the solution to the exact distribution is much more slowly by comparison with the correct problems.

In all above mentioned papers (with the exception of latter one) properly chosen height and frequency discretization as additional information is used (so called "Shachine method"). The statistical information is also in use (Brillet J., 1989). Such approaches do not allow to clear up the true retrieval possibilities. In the paper (Kulikov, 1988) the method of mismatch minimization is used. This method is mathematically quite correct but the retrieved function is supposed to belong to compact functions class. It should be mentioned, that solution in that case can include discontinuities points and, on the contrary, the class of continuous functions, to which the ozone profile must belong, is not compact one.

The mathematically consistent approach to the solution of that problem can be based on the principle of generalized mismatch (Tikhonov, 1983) which uses general a priori information on the smoothness of the exact solution.

### Retrieval method

The relationship between the brightness temperature T<sub>B</sub> of the thermal radio emission and ozone volume mixing ratio profile U(h) at frequency ν and elevation angle θ (after subtracting of other atmosphere components part) can be expressed as

$$T_B(\nu) = \int_0^{\infty} U(h) K(U, \nu, h) dh . \quad (1)$$

where K = K(U, ν, θ, h) is nonlinear kernel of integral equation of the 1-st kind (1). To solve (1) the recurrent algorithm has been worked out, each step of which is based on the generalized mismatch method. On the first step the model ozone profile U<sup>1</sup>(h) from (Longbothum, 1976) is used in the kernel. On the next step the retrieved profile is used.

Let us rewrite (1) in operator form: (2)

$$KU = T_B^{\delta}$$

$T_B^\delta$  is the measured data vector, the

$$\delta \leq \|KU - T_B^\delta\|_{L_2}^2 = \int_{\nu} [T_B(\nu) - T_B^\delta(\nu)]^2 d\nu, \quad (3)$$

where  $T_B$  is the right side of (2)

corresponding to the exact solution  $U(h)$ . The kernel error measure which is related with the absorption coefficient approximation, with the unknown variations in pressure and temperature profiles, and with the nonlinearity, can be evaluated as

$$h = \sup \frac{\|KU - T_B^\delta\|}{\|U\|}. \quad (4)$$

The measurement errors can lead to incompatibility of (1). The measure of incompatibility  $\mu$  satisfies

$$\mu = \inf \|K_h U - T_B^\delta\| \leq \delta + h\|U\|. \quad (5)$$

To find the approximate solution of (2) it is necessary to minimize the functional

$$M^\alpha(U) = \|K_h U - T_B^\delta\|^2 + \alpha \|U\|_{W_2^1}^2 \quad (6)$$

In expressions (3)-(6)  $\|x\|$  designates the norm of the function  $x$  as an element of the spaces of functions  $L_2$  (the space of quadratically summable functions) or  $W_2^1$  (the space of quadratically summable functions which also have generalized derivatives also quadratically summable).

The regularization parameter  $\alpha$  is the root of one-dimensional nonlinear generalized mismatch equation

$$\rho(\alpha) = \|K_h U^\alpha - T_B^\delta\|_{L_2}^2 - (\delta + h\|U^\alpha\|)^2 - \mu^2 = 0, \quad (7)$$

where  $U^\alpha$  is the function that minimizes functional (6).

It is very important that if  $\delta \rightarrow 0$  then  $U^\alpha$  converges to exact solution  $U(h)$  in the  $W_2^1$ -space and, hence, in  $C$ -space (i.e., in the space with the maximum of modulus as a norm). It is the basic preference of the considered method by comparison with others where the convergence is not proved.

The parameters  $h$  and  $\mu$  can be determined in the process of numerical simulation by minimizing of (6). After corresponding discretization the problem of minimization of functional (6) reduces to it's finite difference analogs, which have been well studied as quadratic programming problem. It should be mentioned that within the framework of the method one can easily consider a priori information on the nonnegativeness of

the exact solution. To do this the minimization of (6) must be carried out on the set of nonnegative differentiable functions (minimization with linear restrictions). To that case one can reduce the case in which such a priori information is known as the exact solution is larger (or smaller) of some function. The solution accuracy can be enhanced if there is the possibility to find the solution as a difference from such an model distribution that variations will be located on less height interval comparing with the initial function range.

The recurrence algorithm can be written as

$$K^i U^{i+1} = T_B^\delta, \quad (8)$$

where  $K = K(U^i, \nu, h)$ .

### Results of numerical simulation

Because of the specifics of incorrect inverse problem only on the basis of numerical experiments it is possible to determine the solution accuracy for given measurement errors level as well as to determine the number and position of frequency channels for various  $U(h)$  distributions.

The numerical experiments were performed as follows. The values of  $T_B$  were calculated for given profile  $U(h)$ . A normally distributed error was superposed on these values. The accuracy of the method was evaluated by comparing the retrieved profile with the original  $U(h)$  profile.

The investigations of solution accuracy depending on amplitude, height of maximum and width of ozone profile variations were carried out at the different values of simulated errors using model profile

$$U(h) = U^1(h) + A \exp\left(-\frac{h - h_0}{\Delta h}\right). \quad (9)$$

In the fig.1 one can see the results of retrieval of the ozone profiles (9) with the values of height of maximum  $h_0 = 10, 30, \text{ and } 80$  km. The corresponding values of amplitude were  $A = 5, 15, \text{ and } 5$  ppm, and values of width  $\Delta h = 5, 5, \text{ and } 10$  km. The frequency (shifts from resonance 142.175 GHz) set was: 0.1, 0.2, 0.3, 0.5, 0.7, 1, 2, 3, 5, 7, 10, 20, 50, 100, 200 MHz. The results in line 110 GHz were practically the same. At brightness error level  $\delta T_B = 0.2$  K in the height interval 20 - 50 km the retrieval errors are no more than 1.5% for variations with  $\Delta h > 10$  km. The errors grow up to 10% at  $\Delta h = 5$  km; up to 30% at  $\Delta h = 3$  km. The errors grow also if  $h_0$  is outside above mentioned interval. At  $h_0 = 80$  km as well as at  $h_0 = 10$  km error level is 30%. No more than 3 steps of recurrent algorithm is needed.

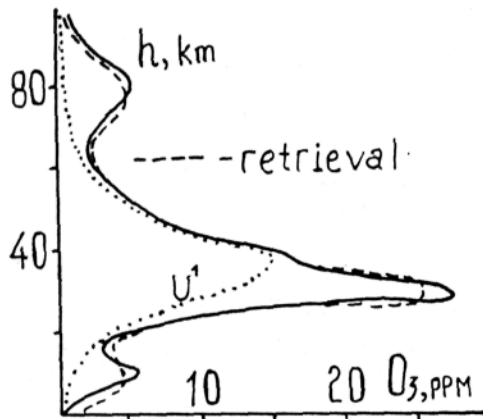


Figure 1

In the fig.2 the case of very narrow variation ( $\Delta h = 1$  km) is shown. One can see that it is very difficult to retrieve such variations not only at  $\delta T_B = 0.2$  K but at  $\delta T_B = 0.02$  K. The retrieval accuracy depends on measurements errors level  $\delta T_B$  not so as incorrect problems. Thus, the retrieval accuracy at  $h_0 = 30$  km and  $\delta h = 10$  km consists 1% at  $\delta T_B = 0.2$  K; 0.25% at  $\delta T_B = 0.02$  K, and 4% at  $\delta T_B = 1$  K.

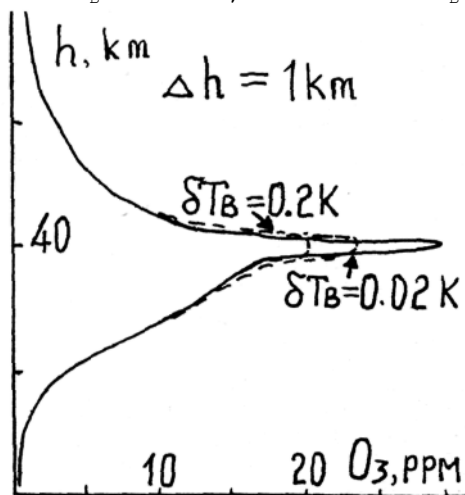


Figure 2.

The retrieval possibilities do not depend very strong on number of frequency channels. The same results in height interval 20-50 km can be obtained with the frequency shifts set: 1, 5, 10, 50, 100 MHz. But absence of near-resonance frequencies does not allow to retrieve the profile at  $h > 50$  km as well as absence of frequencies far from resonance doesn't allow to retrieve the profile at  $h < 20$  km.

The sensitivity of the radiometric method to ozone variations (when the corresponding variations of  $T_B$  are more than errors  $\delta T_B$ ) also has been investigated. At variations with altitude scale  $\Delta h = 10$  km the sensitivity account 0.5-1 ppm/K in height

interval 10-80 km. At  $\Delta h = 3$  km the sensitivity is 1.5-3 ppm/K. That is, at  $\delta T_B = 0.2$  K minimum ozone detectivity would be 0.1 - 0.2 ppm at variations with a height scale 10 km and 0.3 - 0.5 - at  $\Delta h = 3$  km.

### Conclusion

As the most important result of above consideration it is possible to note the new possibilities in ozone profile retrieval, which can be achieved on the basis of the proposed method even at present state of art in radiometry. In particular, it is possible now to investigate the ozone variations in mesosphere. The real restrictions of the ozone retrieval also have been determined.

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