EVOLUTION EQUATION FOR NEAR-FIELD THERMAL RADIO EMISSION[1]

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ABSTRACT

The simultaneous solution of the equations of near-field thermal emission formation and thermal conductivity has been obtained and the expression related the temperature profile of half-space with the measured thermal radio emission (effective brightness temperature) have been obtained. Example of its application in the process of water surface heating the water is presented.

EVOLUTION EQUATION AND RELATED INVERSE PROBLEMS

In the paper [1], on the basis of the simultaneous solution of the equations of radiation transfer and thermal conductivity, the expressions connecting the temperature profile and heat flux dynamics of half-space with the brightness temperature of its thermal radio emission have been obtained. Using these expressions, various methods of radiometry monitoring of the temperature and heat dynamics of water, soils and atmosphere have been developed.

In the case of measurements of the thermal radio emission in the near-field range the formation of the received emission is different. This difference is related to a specific character of the distribution of the quasistationary field component near a radiating surface. The effective depth of the received emission formation appears to be less than the skin-layer depth and depends on the size of the receiver antenna *D* and its height above the medium surface, *h*. This dependence has been obtained from measurements of the emission of a temperature stratified water medium using a specially developed electrically small antenna [2]. It could be considered as a new source of information about the depth temperature distribution [3]. According to [2], the effective brightness temperature T_B of the emission received at $h \geq 0$ above the homogeneous half-space $z \leq 0$ can be written as:

$$T_B(D,h) = \int_{-\infty}^{0} T(z)\left[\iint d^2\kappa F(\kappa,D,h)e^{\gamma(\kappa)z}\right] = \\ = \int_{-\infty}^{0} T(z)\left[\iint_{\kappa\leq k_0} d^2\kappa F_1(\kappa,D)e^{\gamma(\kappa)z} + \iint_{\kappa>k_0} d^2\kappa F_2(\kappa,D,h)e^{\gamma(\kappa)z}\right]dz \quad (1)$$

where $T(z)$ is the temperature depth profile, F, F_1, F_2, γ_1, γ_2 are functions determined in [3]. One can see that the received signal can be expressed in (1) as a sum of the wave (the first term) and the quasistationary (the second term) components. The temperature profile $T(z,t)$ in a medium can be also expressed as the solution of the thermal

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conductivity equation. In the absence of sources at the boundary condition $T(0,t) = T_0(t)$ the temperature profile is determined as

$$T(z,t) = \int_{-\infty}^{t} T_0(\tau)\frac{-z}{\sqrt{4\pi a^2(t-\tau)^3}}\exp(-\frac{z^2}{4a^2(t-\tau)})d\tau\,, \quad (2)$$

where a^2 is the thermal diffusivity coefficient. At the boundary condition $dT(0,t)/dz = -(1/k)J_0(t)$ the corresponding solution is

$$T(z,t) = \int_{-\infty}^{t} J_0(\tau)\frac{-1}{k\sqrt{\pi(t-\tau)}}\exp(-\frac{z^2}{4a^2(t-\tau)})d\tau\,. \quad (3)$$

where J_0(t) is the heat flux through surface $z = 0$ and k is the thermal conductivity coefficient. Substituting (2) and (3) into (1) in the same way as it was done in [1] for far-field measurements and carrying out the necessary transformations, we have the evolution equations for measured effective radiobrightness:

$$T_B(t) = \int_{-\infty}^{t} d\tau T_0(\tau)\iint d^2\kappa\{F(\kappa,D)[\frac{\gamma a}{\pi(t-\tau)} - (\gamma a)^2 erfc(\gamma a\sqrt{t-\tau})e^{(\gamma a)^2(t-\tau)}]\}, \quad (4)$$

$$T_B(t) = -\int_{-\infty}^{t} d\tau J_0(\tau)\iint d^2\kappa[F(\kappa,D)(\gamma a^2/k)erfc(\gamma a\sqrt{t-\tau})e^{(\gamma a)^2(t-\tau)}] \quad (5)$$

These equations can be used to solve correspondence inverse problems, i.e. to obtain boundary conditions $T_0(t)$ and $J_0(t)$ using measurements of $T_B(t)$. It is possible to obtain an exact solution of (4) and (5), and, hence, an exact solution of the problem of temperature profile retrieval as it was done for the case of far-field measurements [1].

However, there is a more straightforward way to solve this inverse problem. It is possible to use the mean value theorem and the condition of the unity normalization of the kernel to obtain from (1) the same equation as for far-field measurements:

$$T_B(D,h) = \int_{-\infty}^{0} T(z)\tilde{\gamma}(D,h)e^{\tilde{\gamma}(D,h)z}dz, \quad (6)$$

where $\tilde{\gamma}(D,h)$ is related with the value of the effective depth of the received near-field thermal emission introduced in [2,3] as $d_{eff}(D,h) = 1/\tilde{\gamma}$. So, using the result obtained in [1], we have the expression for the subsurface temperature profile:

$$T(z,t) = \int_{-\infty}^{t} T_B(\tau)e^{-[z^2/4a^2(t-\tau)]}\left[\frac{1}{\tilde{\gamma}}(\frac{z^2}{2a^2(t-\tau)} - 1) - z\right]\frac{d\tau}{\sqrt{4\pi a^2(t-\tau)^3}}. \quad (7)$$

NEAR-FIELD RADIOMETRY OF WATER TEMPERATURE DYNAMICS

In this study we present our results on retrieval of a water subsurface temperature profile using the contact (h = 0) measured dependence $T_B(t)$ at the antenna size D = 1 cm in (7) in the process of water surface heating (with the help of a wire heater) described in [3]. The results of radiobrightness measurements (for two different antennas with the sizes D = 1 cm and D = 4 cm) are shown in Fig.1 along with contact measured temperature dynamics at the five different depth levels. In Fig.4 one can see that the results of the temperature profiles retrieval are in a good agreement with the contact measured profiles. It is possible to see in Fig.1 the dependence of the radiobrightness on the size of antenna, which is related to the near-field effect, and this dependence is also used [2,3] for temperature profile retrieval. It should be mentioned that the value of D in (1) is determined in practice by the measured value of d_{eff}, so the more simple equation (6) could be used instead of (1) in any of possible applications.

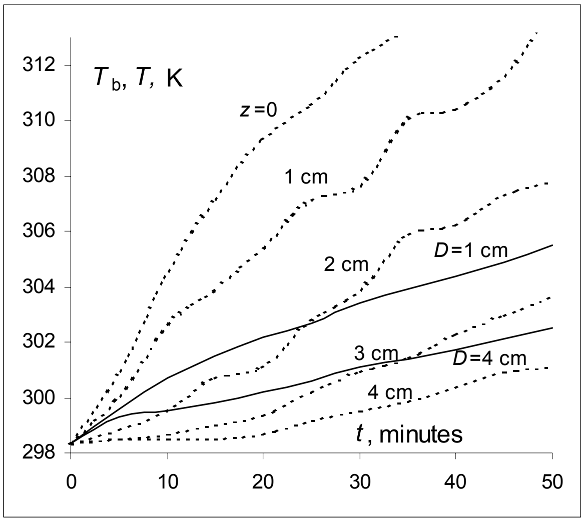

Fig.1. Measurements of the brightness temperature dynamics – solid lines; contact measurements temperature dynamics at different levels inside the water – dashed lines.

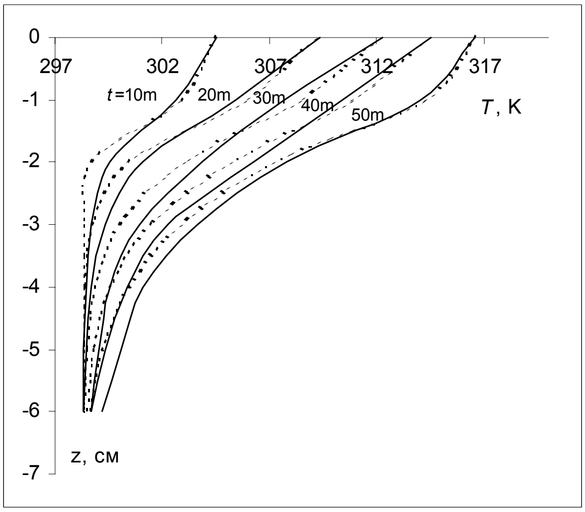

Fig.2. Profiles $T(z)$ retrieved in time interval 10 minutes by measurements of $T_b(t,D=1\text{ cm})$ – dashed lines; profiles $T(z)$ measured by contact thermometer.

CONCLUSION

Expressions are derived that relate the half-space temperature profile and the heat flux with the brightness temperature evolution for the case of near-field measurements. This approach gives a possibility to retrieve subsurface temperature profiles by measured dynamics of the radiobrightness in the same way as by the far-field radiometry measurements.

REFERENCES

1. K.P.Gaikovich, IEEE Trans. Geosci. Remote Sensing, 1994, v.32, *No*.4, p.885.
2. K.P.Gaikovich, A.N.Reznik, JETP Letters, 2000, v.72, *No*.11, p.546.
3. K.P.Gaikovich, A.N.Reznik, V.L.Vaks, N.V.Yurasova. Physical Review Letters, 2002, v.88, *No*.10, p.4302.