Limb-Viewing Refraction Inverse Problem in Duct Case[1]

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Abstract - The limb-viewing refraction inverse problem for retrieval of refraction index with spherical symmetry distribution in the Earth atmosphere is solved as an ill-posed problem, supposing that the retrieval height interval is wider than the height interval for ray perigee in which the refraction is given. The problem is solved for the most important case of duct presence.

Introduction

Limb-viewing refraction measurements have been used for investigation of all the planet atmospheres in the Sun System [1]. Limb-viewing measurements are the measurements of refraction in dependence on the ray perigee height above the planet surface. The corresponding inverse problem consist of the solution of the Abel-type integral equation on the basis of its well-known inverse transformation. The refraction index height profile is determined as the integral of measured refraction dependence.

In the present paper this inverse problem is considered supposing that the retrieval height interval is wider than the height interval for ray perigee in which the refraction is given. Such a formulation leads to an ill-posed inverse problem. There are various possibilities of this problem formulation. The most important case from the practical point of view is the case when the refraction dependence is given from the lower retrieval level up to some determined height level, and the refraction index height profile should be retrieved not only in this layer but also in the height region above this layer. This case is to be considered here for the Earth atmosphere. It is also possible to solve this problem when the refraction dependence is given in two or more height intervals. In all such cases the exact solution of Abel equation is not applicable, and we have the ill-posed inverse problem for integral equation of the 1-st kind.

Problem formulation

For limb-viewing measurements the refraction inverse problem can be expressed by the following integral equation:

$$10^{-6}\int_{r_h}^{r_{max}} \frac{dN}{dh}(r)\frac{-2r}{\sqrt{(nr)^2-(n_h r_h)^2}}dr = \varepsilon(r_h) \quad , \quad r_0 \le r_h \le r_{max} \qquad (1)$$

where $r = r_0 + h$, r_0 is Earth radius, $N = 10^6(n-1)$ is refraction index, n is refractive index, $n_h = n(r_h)$. In the case of duct absence the nonlinear equation (1) can be expressed in a linear form:

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$$10^{-6} \cdot \int_{p_h}^{p_{max}} \frac{dN}{dp}(p) \frac{-2p}{\sqrt{p^2 - p_h^2}} dp = \varepsilon(p_h) \ , \quad p_0 \le p_h \le p_{\max} \qquad (2)$$

$p = nr$, $n_0 = n(r_0)$, $p_0 = n_0 r_0$. In the well-posed case, when the refraction dependence is given on the all the retrieval interval, this equation has the known exact solution:

$$N(p) = 10^6 \cdot \int_{p}^{p_{\max}} \varepsilon(p_h) \frac{dp_h}{\pi\sqrt{p_h^2 - p^2}} \ , \quad p_0 \le p \le p_{\max} \qquad (3)$$

Using the relationship $h = \dfrac{p}{1 + 10^{-6} N(p)} - r_0$ the profile $N(p)$ could be converted into height profile $N(h)$. There is a finest point in this inverse problem, which hasn't been mentioned before. It is easy to see from (1), which depends on derivative of refraction index, that the solution is determined in reality up to arbitrary constant shift. This fact becomes important in the case of numerical solution of (1). To obtain the solution (2), the condition $N(p_{\max}) = 0$ is necessary to use in addition.

Let us consider the equation (1) in the case when its right side is given in the region $0 \le h \le h_1$, $h_1 \le h_{\max}$. The solution of the equation (1) for the region $h_1 \le h \le h_{\max}$ is the typical ill-posed problem, the same type as the astronomical refraction inverse problem in the case of ground-based measurements considered in [2]. It is easy to show that this problem is more complicated than the solution of the Fredholm equation of the 1-st kind. Really, if the refraction index profile $N(p)$ is also considered as known in the region $p_0 \le p \le p_1$, it is easy reduce the problem to Fredholm integral equation of the 1-st kind. To retrieve the refraction index in the region $0 \le h \le h_{\max}$ by refraction measurements in $0 \le h \le h_1$ the equation (1) is solved numerically on the basis of Tikhonov's general discrepancy method [3], which uses the belonging of exact solution to the set of square-integrable functions with square-integrable derivatives. The results of numerical simulation give us the retrieval accuracy at various levels of the refraction error.

Results

In the Fig.1 it is possible to see an example of real typical sonde refraction index profile retrieval by refraction measurements in the region $0 \le h \le 5$ km at the error level $\delta\varepsilon = 5''$ on the basis of linear equation (2). The specific character of the profile above the upper level of measurements ($h_1 = 5$ km) is retrieved with a good quality.

The most interesting is the case, when there is the region of the atmosphere duct above the upper level of refraction measurements. In this region the refraction measurements are absent or distorted because of strong diffraction. Moreover, the linear equation (1) is inapplicable in the presence of duct region. So, it is necessary to use in the solution iteration procedure for nonlinear equation (1).

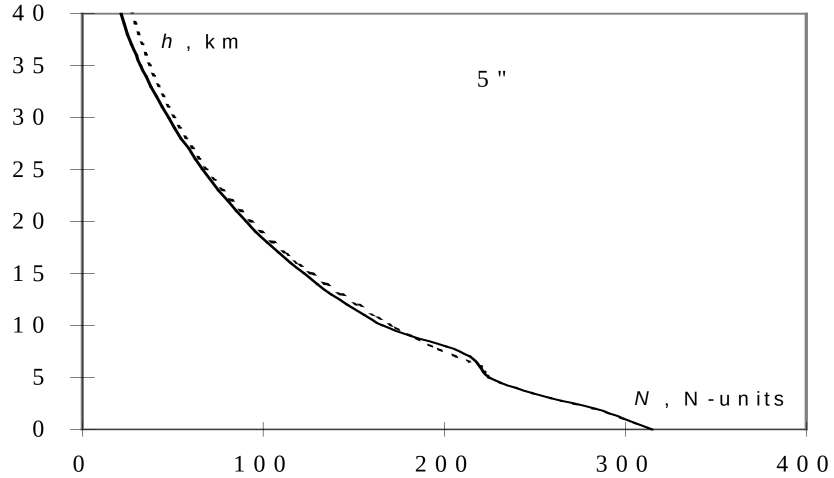

Fig.1.

An example of the retrieval from nonlinear equation (1) in the case of duct stratification is shown in the Fig.2. The duct refraction index distribution if successfully retrieved at $\delta\varepsilon = 15''$.

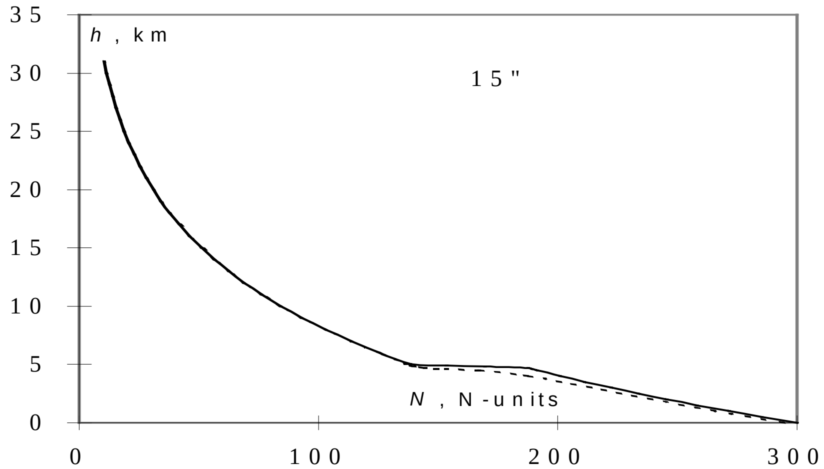

Fig.2.

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