Tikhonov's Algotithm for Two-Dimensional Image Retrieval

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Abstract - Tikhonov's method is successfully applied for the solution of image deconvolution inverse problem in the two-dimensional case. It appeared possible to improve the resolution beyond the aperture limit, and to solve the problem for the many-beam case of synthetic aperture radiometers (SAR) measurements.

Introduction

The problem of the retrieval of true radiobrightness distribution by two-dimensional distribution of measured antenna temperature is very important in radioasrtonomy as well as in remote sensing, especially in the case of SAR measurements. The antenna temperature distribution is a two-dimensional convolution of radiobrightness distribution and antenna pattern as a kernel of the integral. If the kernel is a known function, it is possible to formulate the deconvolution inverse problem to retrieve the true radiobrightness image by measured antenna temperature distribution.

The deconvolution inverse problem consist in the solution of Fredholm integral equation of the 1-st kind, and it is well known that this problem is ill-posed. To solve such a problem it is necessary to use additional (*a priori*) information about the exact solution. This information determines a regularization method. There are various approaches: statistical (maximum entropy) [1], iterative [2], singular systems analysis [3]. In the present paper Tikhonov's method of generalized discrepancy is applied, which uses the common information about the exact solution as a function [4]. It is supposed that the exact solution belongs to the set of square-integrable functions with square-integrable derivatives. The results of numerical simulation give us the retrieval accuracy at various levels of the refraction error.

Problem formulation

The relationship between antenna temperature and radiobrightness distribution can be written as

$$\mathbf{K_h}T_\mathrm{b} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_h(x-s,y-t)\, T_b(s,t)dsdt = T_a^\delta(x,y), \quad (1)$$

where $K_h(w,W)$ is the antenna pattern (kernel), T_a^δ is measured antenna temperature, and $T_b(s,t)$ is the radiobrightness to be found. The measure δ of the error of measured antenna temperature (measurements errors) and measure h of the kernel error satisfy to

$$\|T_a^\delta - T_a\|_{L_2} \le \delta, \qquad \|\mathbf{K} - \mathbf{K}_h\|_{W_2^2 \to L_2} \le h, \tag{2}$$

where T_a corresponds to the exact solution. According to generalized discrepancy method, the approximate solution T_b^α of (1) minimizes the generalized discrepancy functional

$$\mathbf{M}_\alpha[T_b] = \|\mathbf{K}_h T_b - T_a^\delta\|_{L_2}^2 + \alpha\|T_b\|_{W_2^2}^2 \tag{3}$$

at the condition

$$\|\mathbf{K}_h T_b^\alpha - T_a^\delta\|_{L_2}^2 = (\delta + h\|T_b^\alpha\|)^2. \tag{4}$$

The use of Fourier transform permits to obtain the solution for the convolution-type equations in the closed form:

$$T_b^\alpha(s,t) = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\tilde{K}_h^*(\omega,\Omega)\,\tilde{T}_a^{\delta *}(\omega,\Omega)\,e^{i\omega s + i\Omega t}\,d\omega d\Omega}{L(\omega,\Omega) + a[1 + (\omega^2 + \Omega^2)^2]} \tag{4}$$

where $\tilde{K}_h^*(\omega,\Omega) = \tilde{K}_h^*(-\omega,-\Omega)$, $\qquad L(\omega,\Omega) = |\tilde{K}_h(\omega,\Omega)|^2$,

$$\tilde{T}_a^\delta(\omega,\Omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} T_a^\delta(x,y)\,\mathrm{e}^{i\omega x - i\Omega y}\,dxdy, \tag{5}$$

$$\tilde{K}_h(\omega,\Omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_h(u,w)\,\mathrm{e}^{i\omega u - i\Omega \mathrm{w}}\,dudw. \tag{6}$$

The main preference of Tikhonov's method consist of the uniform convergence of the retrieval error to zero at mean square convergence of measurement errors. The closed form of solution (4) and the possibility to use Fast Fourier Transform (FFT) codes permits to make very efficient algorithm. Its accuracy has been investigated in numerical simulations. It appeared possible to retrieve images beyond the aperture resolution limit.

Results

The most interesting results have been obtained in the case of image retrieval by multi-beam SAR data. For more compact presentation, an example was selected, in which two-beam antenna pattern $K_h(w,W)$ coincides with two-modal initial radiobrightness distribution $T_b(s,t)$. (see, in Fig.1). In the Fig.2 the corresponding distribution of antenna temperature distribution (observed image) $T_a(x,y)$ from (1) is given. It is possible to see that the two maxima of true T_b distribution are indistinguishable in observed image T_a.

The approximate solution (4) is shown in Fig.3,4 at two different values of simulated measurement error δ (with respect to T_a, in integral L_2 - space).

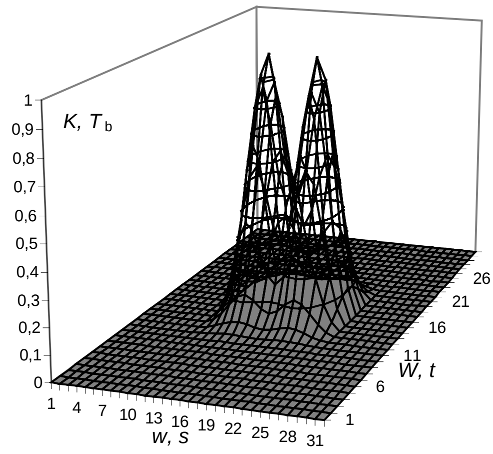

Fig.1. Antenna pattern and initial radiobrightness distribution, in conventional units

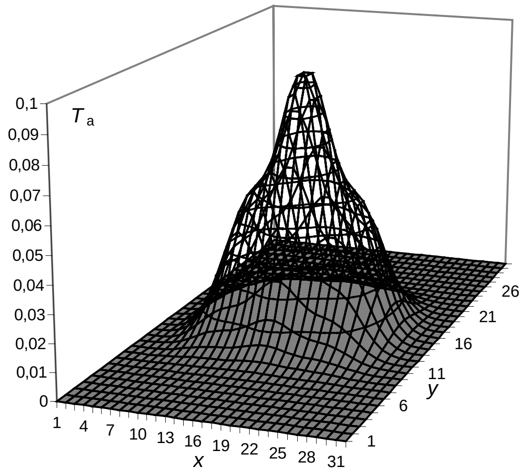

Fig.2. The corresponding observed image.

The Tikhonov's method of image retrieval permits to distinguish two-modal structure of initial distribution even at comparatively low measurement accuracy.

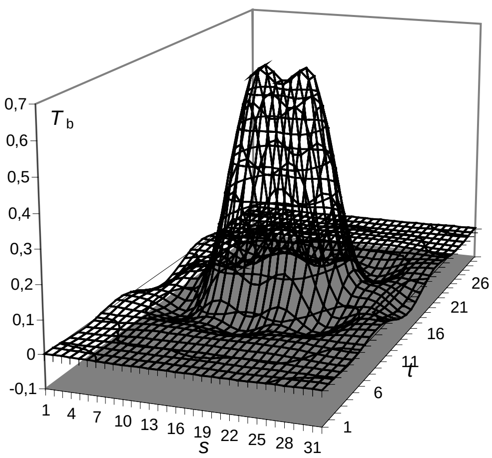

Fig.3. Retrieved image at error level 1%.

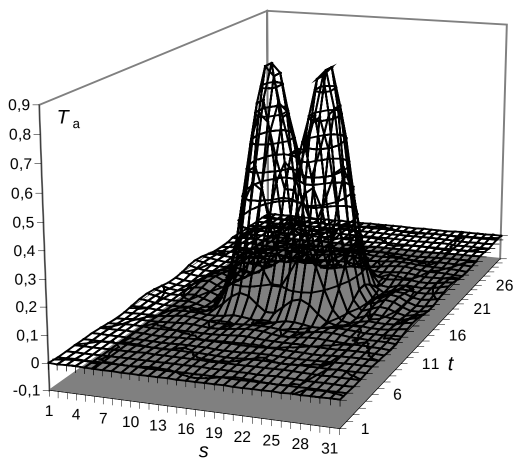

Fig.4. Retrieved image at error level 0.01%.

Along with error decrease, the retrieval error converges to zero, and it appears possible to retrieve true radiobrightness distribution beyond the aperture resolution limit even in the case of many-beam SAR measurements. One can see that the initial distribution in Fig.1 and retrieved image in Fig.4 are very similar.

References

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