Technique for Determination of RF-Magnetic Field Dependence of the HTS Surface Impedance by Microwave Resonators[1]

K.P.Gaikovich[1], A.N.Reznik[2]

[1] *Radiophysical Research Institute, B.Pecherskaya st., 25/14, 603600, N.Novgorod, Russia.*

[2] *Institute for Physics of Microstructures of RAS, GSP-105, 603600, N.Novgorod, Russia.*

Introduction. The investigations of nonlinear electromagnetic properties of high-temperature superconductors (HTS) in the last years offer raised interest in connection with prospects of HTS application in various microwave devices (resonators, filters, antennas etc.), with problems of quality control of these materials, with fundamental problems of physics of superconductors. Nonlinearity of HTS is usually characterized by dependence of a surface impedance Z_S from the amplitude of a variable magnetic field H on HTS surface, i.e. $Z_S(H)$. These dependence is determined by using of microwave resonators of various designs [1-5]. Measuring parameters are: nonlinear broadening of the frequency response Δf_B and the resonant frequency shift Δf_0. The algebraic equations which connect $\Delta f_B, \Delta f_0$ with $R_S(H) = \mathrm{Re}\, Z_S(H),\ \ X_S(H) = \mathrm{Im}\, Z_S(H)$ are in use. For all resonators types there is a strong inhomogeneity of a field H distribution on a HTS surface. In the given work it is shown that HTS nonlinearity and fields inhomogeneity leads to the essential errors of convenient techniques, and a new approach to a problem of diagnostics of nonlinear microwave properties of HTS is advanced.

The integral equations. The techniques, used for determination of $Z_S(H)$, are based on the next equation:

$$\Delta f = \Delta f_0 + (i/2)\Delta f_B = (i/8\pi W)\int_S H^2 Z_S d^2 r \quad , \tag{1}$$

where W is the energy, stored into the resonator, and the integration is made on a HTS surface. In the nonlinear resonator the nonuniform field structure leads to inhomogeneous distribution of $Z_S(H)$ on a HTS surface. In this case we obtain from (1)

$$\Delta f = (i/2)G < Z_S > \quad , \tag{2}$$

where $G = (1/4\pi W)\int_S H^2 d^2 r$ - geometrical factor, which is calculated for each particular resonator or is measured by calibration,

$$< Z_S >= \int_S H^2 Z_S(H) d^2 r \Big/ \int_S H^2 d^2 r \tag{3}$$

averaged surface impedance. Thus, the use of the equation (2), as it is done in [1-5], yelds $< Z_S >$ rather than Z_s, which largely reduces the value of obtained results, since $< Z_S >$ depends not only on properties of HTS material, but also on the resonators type and the excited mode.

We shall consider, that the resonator contains one HTS film as a conducting wall, which dependence $Z_S(H)$ is the sought-for parameter. The H field structure on a HTS surface near resonant frequency is determined by own function of the appropriate mode $\Phi(\vec{r})$:

$$H(\vec{r}) = H_m \Phi(\vec{r}) \quad , \tag{4}$$

[1] This work was supported by RFBR under grant 96-02-16997 and by Russian State Program on Physics of Condensed Matters under grant 96129.

where H_m is the maximum value of H (Φ is normalized so that $\Phi_{\max} = 1$). The field H_m of the nonlinear resonator for each value of the input power is calculated on the basis of well known techniques (for example, for the microstrip resonator, see [6]). Passing in (3) from integration on coordinate variable to integration on H, with the account (4), we obtaine

$$< Z_S(H) >= \int_0^{H_m} K(H_m, H) Z_S(H) dH \ , \qquad (5)$$

where $K(H_m, H)$ is the kernel of the integral equation, which depends on the resonator type and the exited mode ($\int_0^{H_m} K(H_m, H) dH = 1$). We have received the expressions for $K(H_m, H)$ for different resonators types: confocal, microstrip, cavity, dielectric.

Using (5), we have calculated a relative excess $R_S(H)$ over $< R_S(H) >$ for various types of resonators and for typical dependencies $R_S(H)$, observable in experiments [1-5]. We have received, that the use of the equation (2) gives underestimated value of $R_S(H)$ in 1.3-1.7 times less for cavity-, in 2-3 times lessfor confocal-, in 4.5-7 times less for microstrip resonators.

Method of solution of the inverse problem. The method offered us consists in measurement of $< Z_S >$ by the HTS resonator using the formula (2) at several input powers. Then $Z_S(H)$ is defined by solution equation (5). Equation (5) is the integral equation of Volterra of the 1-st kind, whose solution is an ill-posed inverse problem. In the given work the Tikhonov method [7] was applied for solution of (5). We shall rewrite the equation (5) in the operator form

$$\hat{K}R = R_m^{\delta} \ , \qquad (6)$$

where R_m^{δ} - vector of experimental datum, received with some tool error. In the Tikhonov method the approached solution is searched by minimization of functional

$$M^{\alpha}(R) = \left\| \hat{K}R - R_m^{\delta} \right\|_{L_2}^2 + \alpha \| R \|_{W_2^1}^2 \ , \qquad (7)$$

where $\| f \|_{l_2}^2$ is the norm of function f in space L_2 of square integrable functions, and $\| f \|_{W_2^1}^2$ - norm of function f in the space W_2^1 of square integrable functions together with their derivatives. The problem of minimization of functional (7) after appropriate discretization was solved by the method of conjugate gradients [8]. Smoothing of the solution received from (7) is adjusted by parameter α, which, as it is shown in [7], is connected with an integrated measure of an inaccuracy of experimental data and is sought as the root of the nonlinear algebraic equation

$$\rho(\alpha) = \left\| \hat{K}R^{\alpha} - R_m^{\delta} \right\|_{L2}^2 - \delta^2 \ , \qquad (8)$$

where R^{α} is the solution of (7).

Parameter of an effective error δ in (8) includes all errors of measurements and interpretations. Thus, in a used method the smoothing of the solution is determined by an error measure δ. At high measurements accuracy, the error δ reduces, and, hence, there is less smoothing in the obtained solution, i.e. the details of $Z_S(H)$ can be reconstructed.

Results of numerical modelling. The investigation of opportunities of retrieval is carried out on the basis of numerical simulation for typical $R_S(H)$ and limits of tool errors. We have used the expression for $K(H_m, H)$ in (5) obtained for a rectangular cavity resonator and the power field dependence for R_S, which is met more often in practice. The modeling of reconstruction procedure was done in the following closed circuit. For the given initial function $R_S(H)$ exact dependence $< R_S(H_m) >$ was calculated from (5), for which a random error with the given rms δR_m was added in discrete points $m = 1,2...M$, simulating the measurements errors. Received thus "data of measurements" were used for solution of the inverse problem and the retrieved dependencies were compared with the initial one. For estimation of the efficiency of the Tikhonov method equation (5) was solved also by the method of the direct inversion, i.e. by solving a numerical analogue of the integral equation (5), which after appropriate discretization becomes a linear system of algebraic equations.

The results of numerical modeling are presented in Fig.1,2. In Fig.1 one can see an example of retrieval. In Fig.2 the normalized dependencies of an integral measure of an inaccuracy $\delta R = [H_{\max}^{-1} \int_0^{H_{\max}} \sigma_R^2(H) dH]^{1/2}$ on a number of experimental points M are shown. It was obtained from the results of numerical simulation that the Tikhonov method provides qualitative reconstruction at $\delta R_m \le 0.02 mOhm$, whereas the direct inversion - only at errors $\delta R_m \le 0.005 mOhm$, close to extreme values, achievable for the state-of-art measurement techniques. We emphasize the existence of optimum value M, at which the error of reconstruction is minimum (Fig.2), and this minimum for the Tikhonov method corresponds to a much smaller error than for a method of the direct inversion, and at smaller values of M.

Conclusions. The method offered here is based on the theory of the solution of ill-posed inverse problems for $Z_S(H)$, which allows to take into account the inhomogenneous structure of an electromagnetic field in the resonators.

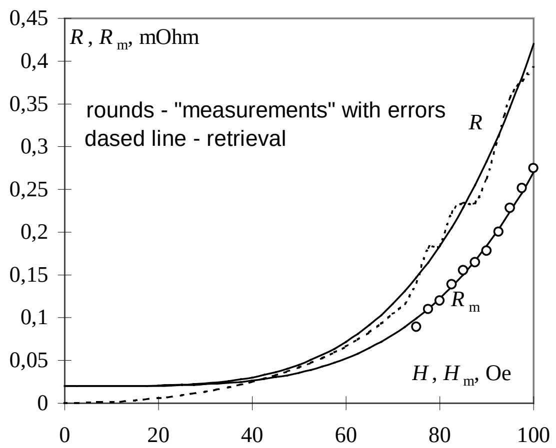

Fig1.

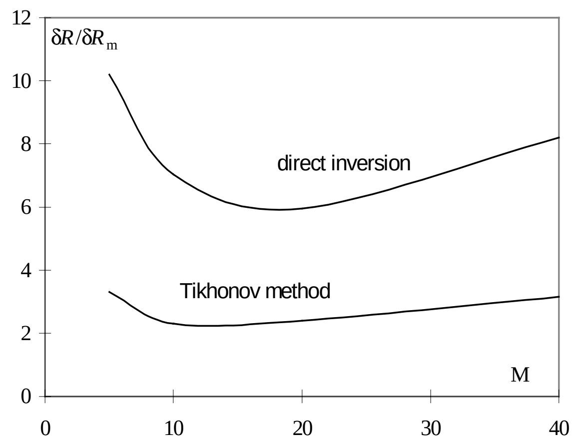

Fig.2.

References

1. Oates D.E. et al. J. Supercond. 1990, v.3, N3, p.251.
2. Cooke D.W. et al. J. Supercond. 1990, v.3, N3, p.261.
3. Shen Z.-Y. et al. IEEE Trans. Microwave Theory Techn. 1992, v.40, p.2424.
4. Golosovsky M.A. et al. Phys. Rev. B. 1995, v.51, p.6462.
5. Hein M. et al. IEEE Trans. Appl. Supercond. 1997, v.7, p.1564.
6. Sheen D.M. et al. IEEE Trans. Appl. Supercond. 1991, v.1, p.108.
7. Tikhonov A.N. et al. Regularization algorithms and a priori information. M.: Nauka, 1983.
8. Vasil’ev F.P. Numerical methods for solution of extreme problems. M.: Nauka, 1981.