NATIONAL ACADEMY OF SCIENCES OF UKRAINE

SCIENTIFIC COUNCIL OF NATIONAL ACADEMY OF UKRAINE ON THE PROBLEM "PHYSICS AND ENGINEERING OF MILLIMETER AND SUBMILLIMETER ELECTROMAGNETIC WAVES"

INSTITUTE OF RADIOPHYSICS AND ELECTRONICS OF NATIONAL ACADEMY OF SCIENCES OF UKRAINE

.

INSTITUTE OF GENERAL PHYSICS OF RUSSIAN ACADEMY OF SCIENCES

MOSCOW STATE UNIVERSITY NAMED AFTER M.V.LOMONOSOV

INTERNATIONAL ASSOCIATION "MICROWAVES"

INTERNATIONAL KHARKOV SYMPOSIUM "PHYSICS AND ENGINEERING OF MILLIMETER AND SUBMILLIMETER WAVES"

Kharkov, 7-10 June 1994

CONFERENCE PROCEEDINGS
Volume III

Kharkov
Institute of Radiophysics and Electronics
of National Academy of Sciences of Ukraine 1994

THE SIMULTANEOUS SOLUTION OF EMISSIOH TRANSFER AND THERMAL CONDUCTIVITY EQUATIONS IN STRATIFIED HEDIA

K.P. Gaikovich
Radiophysical Research Institute
Nizhny Novgorod

The inverse problem of the temperature profile retrieval in half-space z≤ 0 is based on the well-known emission transfer equation :

$$T_B(\lambda) = \int_{-\infty}^{0} T(z)\ B(z,\lambda)\ dz\ . \qquad (1)$$

where T(z) ie the temperature profile, T_B is the brightness temperature of the thermal radioemission, λ. - wavelength. The Eq.(1) is an incorrect Fredholm integral equation of the 1-st kind and additional *a priori* information is necessary for Its solution. This information may be statistical one (interlevel covariance), or it may be the information on smoothness of exact solution (Tikhonov's methods).

It is also possible to take into account, that T(z) satisfies the thermal conductivity equation. The general approach to the solution of the problem in the ease of one-dimensional inhomoheneous media can be based on the use of Duamel integral for thermal conductivity equation:

$$\frac{\partial T}{\partial t}(z,t) = LT, \qquad (2)$$

where L is a linear differential operator which can contain the derivatives and functions of z. If one knows the solution $T^1(z,t)$ of (2) with boundary conditions

$$\alpha T^1(0,t) + \beta\frac{\partial T^1}{\partial z}(0,t) = \mathbf{1}(t), \qquad (3)$$

α and βare constants, **1**(t) = 1, t ≥ 0; **1**(t) = 0, t < 0, the solution of (2) with boundary conditions

$$\alpha T(0,t) + \beta\frac{\partial T}{\partial z}(0,t) = b(t), \qquad (4)$$

can be written as

$$T(z,t) = \frac{\partial}{\partial t} \int_{-\infty}^{t} T^1(z,t-\tau)\, b(\tau)\, d\tau , \quad z < 0. \tag{5}$$

The substitution of (5) in (1) gives:

$$T_B(t) = \int_{-\infty}^{0} \left[\frac{\partial}{\partial t} \int_{-\infty}^{t} T^1(z,t-\tau)\, b(\tau)\right] B(z,\lambda)\, dz . \tag{6}$$

The expression (6) can be written as:

$$T_B(t) = \frac{\partial}{\partial t} \int_{-\infty}^{t} b(\tau) \left(\int_{-\infty}^{0} T^1(z,t-\tau)\, B(z,\lambda)\, dz\right) d\tau . \tag{7}$$

Denoting

$$T_B^1(t) = \int_{-\infty}^{0} T^1(z,t)\, B(z,\lambda)\, dz , \tag{8}$$

the final equation can be obtained:

$$T_B(t) = \frac{\partial}{\partial t} \int_{-\infty}^{t} b(\tau)\, T_B^1(t-\tau)\, d\tau = \int_{-\infty}^{t} b(\tau) \frac{\partial}{\partial t} T_B^1(t-\tau)\, d\tau . \tag{9}$$

One can see, that if we know the response $T^1_B(t)$ of the brightness temperature by boundary condition (1), it is possible to solve (9) relative b(t) and to determine subsurface temperature profile from (2). If we have simultaneous measurements of $T_B(t)$ and b(t) is possible to solve (9) relative $d/dt\, T^1_B(t)$. Then we can use this function as a kernel in (9) to solve this equation relative to b(t). In such approach there is no need in knowing of subsurface dielectric profiles, and only to determine the subsurface temperature profile according (5) thermal conductivity profile must be used. It is also possible to determine $T^1(z,t)$ and $d/dt\, T^1_B(t-\tau)$ on the basis of numerical calculations for given thermal and dielectrical structure of the medium. In the case of homogeneous medium expressions for $d/dt\, T^1_B(t-\tau)$ and inversion of (9) have been obtained in /1,2/.

References

1. K.P.Gaikovich, A.N.Reznic, R.V.Troitsky. Digest of IGARSS-91, 3, 1991.
2. 2. K.P.Gaikovich. Izvestija vuzov.Radiofisika, 1993, 36, N1, 16.