ULTRA LOW FREQUENCY SOUNDING AND TOMOGRAPHY OF EARTH CRUST

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Abstract

Methods of the ultra low-frequency electromagnetic sounding of the conductivity into the earth crust based on wideband (in the frequency range 0.001-1000 Hz) measurements of electromagnetic fields using both natural and artificial sources of are considered. The inverse problem is solved to retrieve continuous one-dimensional profiles of conductivity using iterative approach for the non-linear integral equation derived from Maxwell's equations. A tomography method that uses many-frequency data of the 2D electromagnetic scanning along the earth surface is also proposed. The retrieval of the 3D structure of the conductivity in this method is based on the solution of the 3D integral equation that can be reduced to the convolution equation with respect to lateral co-ordinates in the frameworks of Born approximation.

Keywords: electromagnetic sounding, tomography, earth crust, conductivity

1. INTRODUCTION

This problem of ultra low-frequency electromagnetic (ULF) geomagnetic sounding of earth crust has been formulated firstly by A.N.Tikhonov [1] and solved by him for the one-dimensional multi-layered media. The frequency dependence of measured fields is in use in this sounding because the effective depth of the signal formation (skin-depth) is frequency-dependent and achieves few kilometers at lowest frequencies. This method is known as magnetotelluric exploration.

An iteration method of the solution of this inverse problem for continuous one-dimensional distributions of the earth conductivity (profiles of conductivity) has been proposed in [2], and results of numerical study of this method are presented in this paper.

It is clear, however, that the distribution of conductivity in the earth crust is, in general, three-dimensional (3D), so methods should be worked out to solve this 3D problem of the earth crust tomography. In this paper, a tomography method, based on the solution of 3D integral equations that are the 2D convolution equation with respect to lateral co-ordinates is developed. This method uses 2D data of near-surface measurements of fields' variations in dependence on frequency (a parameter that determines the effective depth of signal formation into a medium).

2. ITERATION METHOD OF CONDUCTIVITY PROFILING

In the ULF range the approximation of Leontovich's boundary conditions is valid, so the field in the medium can be considered as a plane wave with components E_x, H_y of electric and magnetic field respectively that propagate in the nadir direction relative the earth surface. Also, the permittivity at low frequencies is determined by the conductivity σ as

$$\varepsilon = \varepsilon' - i\varepsilon'' \approx -i\frac{4\pi\sigma}{\omega}, \qquad (1)$$

where ω is the cyclic frequency. Fields are measured at the surface level in dependence on frequency (indexes are omitted):

$$E(z=0,\omega) = E_0(\omega), \qquad (2)$$
$$H(z=0,\omega) = H_0(\omega).$$

These data can be compared to frequency dependences for the homogeneous medium with the conductivity $\sigma(z) = \sigma(0) = \sigma_0$:

$$E^0(\omega,z) = E_0^0(\omega)\exp(\frac{z}{\delta} + i\frac{z}{\delta}), \qquad (3)$$
$$H^0(\omega,z) = \frac{c(i-1)}{\omega\delta}E^0 = H_0^0(\omega)\exp(\frac{z}{\delta} + i\frac{z}{\delta}),$$

where $\delta = \dfrac{c}{2\pi\omega\sigma_0}$ is the skin-depth. To solve the problem of the conductivity profile sounding, it is possible to obtain from the Maxwell's equations the integral expression [2]:

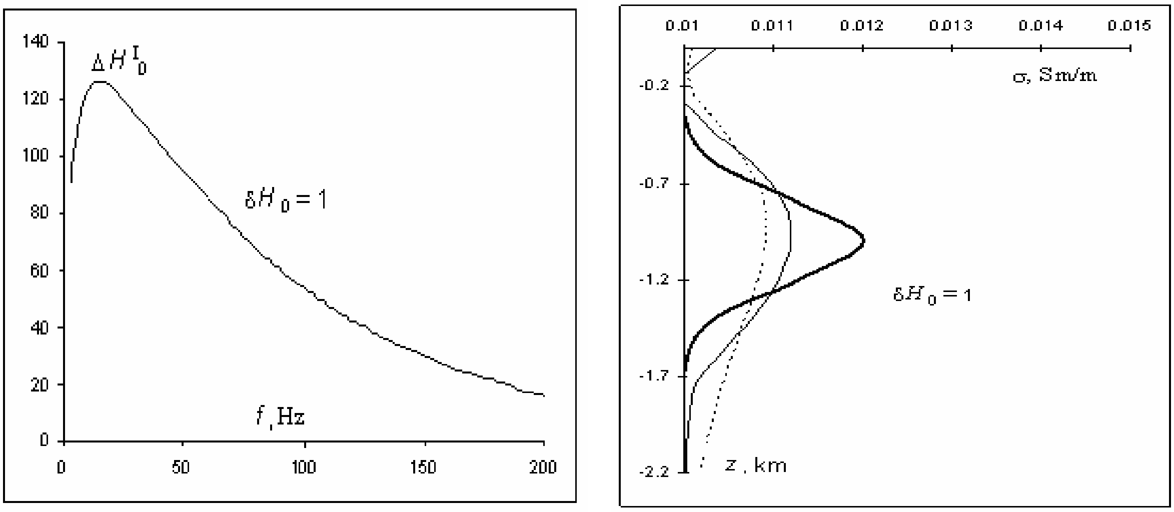

Fig. 1. An example of numerical modeling of the retrieval of the profile of conductivity. Left, "measured data" in arbitrary units in dependence on $f = 2\omega/\pi$ at the *rms* of the random error $\delta H_0 = 1$; right, initial profile (bold), retrieval results: 1st iteration (dashed), 2nd iteration (solid).

$$H_0(\omega) = \frac{4\pi i\omega}{c^2}\int_{-\infty}^{0}\left[\int_{-\infty}^{z'} H(\omega, z'')dz''\right]\sigma(z')dz' . \quad (4)$$

Using (3) as the first guess ($H_1 = H^0$) that is, in fact, the Born approximation, it is possible to solve the non-linear integral equation (4) iteratively:

$$\Delta H_0^{R,I}(\omega) = \int_{-\infty}^{0} K_i^{R,I}(\omega, z')\Delta\sigma_{i+1}(z')dz' , \quad (5)$$

$$K_i^{R,I}(\omega, z') = \mp\frac{4\pi\omega}{c^2}\int_{-\infty}^{z'} H_i^{I,R}(z'', \omega)dz'' ,$$

where $\Delta H_0 = H^0 - H_0$, $\Delta\sigma = \sigma(z) - \sigma_0$ and the expression (5) is written for real and imaginary parts of the field complex amplitude. At each step of the iteration, the expression (5) can be solved as a Fredholm integral equation of the 1st kind using the Tikhonov's method of generalized discrepancy [1].

In Fig.1 one can see results of the numerical modeling of the retrieval of conductivity profile. One can see that it is possible to retrieve the subsurface profile of the conductivity using the proposed method with the high enough accuracy. The number of useful iterations is depended on the accuracy of measurements.

3. Scanning ULF Tomography

It is obvious that in general case the earth conductivity structure is three-dimensional. Let a scattering region with the complex permittivity $\varepsilon(\mathbf{r}, \omega) = \varepsilon_0(\omega) + \varepsilon_1(\mathbf{r}, \omega)$ is embedded in the half-space $z \le 0$, where $\varepsilon = \varepsilon_0(\omega)$. In the presence of the scattering region, the electric field $\mathbf{E}(\mathbf{r})$ is determined as

$$\mathbf{E}(\mathbf{r}) = \mathbf{E_0}(\mathbf{r}) + \frac{1}{4\pi\varepsilon_0}\int_V \overleftrightarrow{\mathbf{R}}(\varepsilon_1, \mathbf{r}, \mathbf{r}')\mathbf{E_0}(\mathbf{r}')d\mathbf{r}' \quad (6)$$

where the reference (unperturbed) field $\mathbf{E_0}(\mathbf{r})$ is relates to an external source. The resolvent $\overleftrightarrow{\mathbf{R}}$ is determined by $\varepsilon_1(\mathbf{r})$ and the Green tensor as known Neumann series.

Unfortunately, it is difficult to use the integral expression (6) because it is uneasy task to solve such 3D nonlinear equations. The possible consideration of the tomography problem can be based on the Born approximation ($\varepsilon_1(\mathbf{r}) << \varepsilon_0$), where $\mathbf{E}(\mathbf{r}) \approx \mathbf{E_0}(\mathbf{r}) + \mathbf{E_1}(\mathbf{r})$ and the scattered field components of electric and magnetic fields are determined as:

$$\mathbf{E}_1(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \varepsilon_1(\mathbf{r}')\overleftrightarrow{\mathbf{G}}(\mathbf{r} - \mathbf{r}')\mathbf{E_0}(\mathbf{r}')d\mathbf{r}' . \quad (7)$$

$$\mathbf{H}_1(\mathbf{r}) = \frac{i\omega}{4\pi c}\int_V \varepsilon_1(\mathbf{r}')\overleftrightarrow{\mathbf{G}}_H(\mathbf{r} - \mathbf{r}')\mathbf{E_0}(\mathbf{r}')d\mathbf{r}'$$

where $\overleftrightarrow{\mathbf{G}}_{E,H}$ are known Green tensors. For the plane wave reference fields (3) in the medium, equations (7) are the 2D convolution equations over lateral coordinates. So, it appears possible to work out the tomography method based on of 2D Fourier transform of (7) that reduces this problem to the solution of one-dimensional Fredholm integral equations of the 1-st kind relative to the depth profile of lateral spectral components of permittivity variations σ_1. Taking into account (1), obtain:

$$\mathbf{E}_1(\kappa_x,\kappa_y,\omega)=\frac{1}{8\pi}\frac{E_0}{\sigma_0}\int_{-\infty}^{0}\sigma_1(\kappa_x,\kappa_y,z')\cdot$$
$$\cdot\exp(\frac{z'}{\delta}+i\frac{z'}{\delta})\cdot\exp[\pm i\sqrt{k^2-\kappa_\perp^2}(z-z')]\cdot$$
$$\cdot\frac{(k^2-\kappa_x^2)\vec{x}_0-\kappa_x\kappa_y\vec{y}_0\mp\kappa_x\sqrt{k^2-\kappa_\perp^2}\;\vec{z}_0}{i\sqrt{k^2-\kappa_\perp^2}}dz'$$

$$\mathbf{H}_1(\kappa_x,\kappa_y,\omega)=H_0\frac{(1+i)\omega\delta}{4c^2}\int_{-\infty}^{0}\sigma_1(\kappa_x,\kappa_y,z')\cdot$$
$$\cdot\exp(\frac{z'}{\delta}+i\frac{z'}{\delta})\cdot\exp[-i\sqrt{k^2-\kappa_\perp^2}(z-z')]\cdot$$
$$\cdot\frac{\vec{y}_0\sqrt{k^2-\kappa_\perp^2}-\vec{z}_0\kappa_y}{i\sqrt{k^2-\kappa_\perp^2}}dz' \tag{8}$$

where $\kappa_\perp^2=\kappa_x^2+\kappa_y^2$, $k^2=(\omega/c)^2\varepsilon_0$, $\sigma_1(\kappa_x,\kappa_y,\omega)=\frac{1}{4\pi^2}\iint\sigma_1(x,y,x)\exp(-i\kappa_x x-i\kappa_y y)dxdy$ is the lateral spectrum of conductivity variations, $\mathbf{H}_1(\kappa_x,\kappa_y,z)$ is the lateral spectrum of measured variations of the magnetic field at the surface level (z = 0).

If, for example, variations of the *y*-component of the magnetic field are measured at the surface level in dependence on frequency, one has from (8) yet more simple equation for the lateral spectrum:

$$H_{1y}(\kappa_x,\kappa_y,\omega)=H_0\frac{(1+i)\omega\delta}{4c^2}\int_{-\infty}^{0}\sigma_1(\kappa_x,\kappa_y,z')\cdot$$
$$\cdot\exp(\frac{z'}{\delta}+i\frac{z'}{\delta})\cdot\exp[-i\sqrt{k^2-\kappa_\perp^2}z']\cdot dz' \tag{9}$$

It is possible to see from (9) that the contribution of medium inhomogeneities in field variations is decreased at the scale of the skin-depth δ that depends on frequency. But there is also another kind of decay for near-field (evanescent) components (at $\kappa_\perp^2>|k|^2$) of the received signal. These components are rapidly decreased nearby inhomogeneities.

The expression (9) is a Fredholm integral equation of the 1st kind relative to the depth profile of the lateral conductivity variations spectrum for each pare of lateral wavenumbers. It is an ill-posed problem, and proper regularization methods should be applied for its solution. The Tikhonov's method of generalized discrepancy [1] is very suitable to solve this one-dimensional problem, because the only parameter of this method – the integral error for the lateral spectrum of measurement data – is easily estimated by the corresponding integral error of measured signal using the Plansherel's theorem.

Finally, the 2D inverse Fourier transform over lateral co-ordinates of the retrieved lateral spectrum of the conductivity gives us the desired 3D distribution of the conductivity into the earth crust:

$$\sigma_1(x,y,z)=\iint\sigma_1(\kappa_x,\kappa_y,z)\exp(i\kappa_x x+i\kappa_y y)d\kappa_x d\kappa_y$$

To realize this tomography method, it is enough to make measurements only in two places simultaneously because, in fact, the equation (9) depends on the ratio of the reference and scattered fields. The first place (the reference point) should be chosen somewhere outside the inhomogeneous region; the second is one of 2D grid points of the region to be studied. So, step by step, all 2D data can be obtained.

4. CONCLUSION

The possibilities of ULF electromagnetic sounding of the subsurface conductivity of the earth crust are studied. Cases of one-dimensional and three-dimensional conductivity inhomogeneities are considered.

In the first case, an iteration method based on the solution of non-linear integral equation has been worked out to retrieve the subsurface profile of conductivity. Results of numerical modeling are presented.

A tomography method is proposed that uses 2D wide band ULF measurements of near-surface field variations above the studied region of 3D subsurface inhomogeneities of earth conductivity. Method uses the lateral Fourier transform of the integral expression for the scattered field obtained in the Born approximation. This transform reduces the 3D integral equation to one-dimensional Fredholm integral equation of the 1st kind relative to the depth profile of the lateral spectrum of conductivity. The inverse 2D Fourier transform of this equation solution gives the desired 3D distribution of the subsurface conductivity.

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