MULTIFREQUENCY MICROWAVE TOMOGRAPHY OF ABSORBING INHOMOGENEITIES

[1] Gaikovich K. P., [1] Gaikovich P. K., [2] Maksimovitch Ye. S., [2] Badeev V. A.

[1] Institute for Physics of Microstructures RAS, GSP-105, Nizhniy Novgorod, Russia
E-mail: gai@ipm.sci-nnov.ru
[2] Institute of Applied Physics National Academy of Sciences of Belarus, Minsk, Belarus
E-mail: makhel@iaph.bas-net.by

Abstract

A method of the near-field scanning coherent tomography has been developed for the microwave diagnostics of the 3D subsurface structure of the complex permittivity. This method uses data of 2D scanning over lateral co-ordinates above the ground surface with the dipole emitter-receiver system. Multifrequency measurements provide the depth sensitivity. The regularization algorithm based on the Tikhonov's method of generalized discrepancy has been developed for Fredholm integral equations of the 1-st kind with complex-valued functions and applied for the solution of the corresponding inverse scattering problem. Results are presented for the tomography of 3D complex permittivity distribution originated by the melting ice sample buried in the sand.

Keywords: microwave subsurface sounding, near-field scanning tomography, inverse scattering problem, complex permittivity

1. INTRODUCTION

Methods of active and passive electromagnetic subsurface sounding are widely in use to determine the inner structure of permittivity or temperature distribution inside various media [1]. Tomography methods (retrieval of 3D structures) lead to the most complicated inverse problems. In the active electromagnetic tomography, the 3D permittivity distribution of subsurface inhomogeneities should be obtained by measurements of the scattered field. To obtain the necessary data set, the 2D transversal distribution of the scattered field should be measured by the 2D scanning along the media interface in dependence on a third (depth-sensitive) parameter, such as frequency. The corresponding inverse scattering problem is based on the solution of the 3D nonlinear integral equation of the 1-st kind with the 6D kernel [2]. It is clear that straightforward methods of solution lead to hard restrictions on achievable resolution. For far-field measurements there is also the known Rayleigh limitation of resolution.

In the considered here method of coherent microwave near-field tomography of the subsurface permittivity, above-mentioned difficulties are surmounted. We use here the general approach to the near-field scanning tomography [2] and its application to the near-field electromagnetic scattering [3-4] based on 2D lateral plane wave decomposition of corresponding Green functions that reduces the initial 3D integral equation to the one-dimensional Fredholm integral equation of the 1st kind relative to the depth profile of the lateral spectrum of permittivity. This approach overcomes problems of the solution of 3D integral equations and leads to a high-performance and mathematically consistent algorithm based on the method of generalized discrepancy [5].

First experimental results of this method of tomography has been presented in [6]. Measurements of the scattered signal for an ice target buried in the sand have been carried out using experimental set-up including vector network analyzer Agilent E5071B, two identical bow-tie antennas in bi-static configuration, operating in the frequency range of 1.7 to 7.0 GHz and the sandbox. The frequency serves here as the depth- sensitivity parameter. In order to form the necessary 3D data set, C-scan has to be obtained. This is achieved by collecting a series of A-scans (801 points over the frequency range) on a horizontal survey lines

978-1-4244-7468-4/10/$26 ©2010 IEEE

(11×15 measuring points through 2 cm) over the sandbox surface.

The theory of this tomography method, developed in [2-3], involves in the integral equation the distribution of the complex permittivity, but, in the beginning of our experiments, algorithms were available for only real-value integral equations. Because of this reason, the theory has been modified in [6] for the tomography of real-value targets, and the known algorithm of Tikhonov's method of generalized discrepancy has been applied to retrieve the 3D distribution of the real part of permittivity.

However, permittivity perturbations related to the melting ice include also the absorbing range related to the water diffusion around the target, and it was the reason to develop necessary algorithms for complex-value integral equations and to use them in analysis. These algorithms have been developed in [5] and studied in the numerical simulation [7] for conditions of this experiment.

2. THEORY

Let us the scattering range is embedded in a medium with the complex permittivity ε_0, so, its total 3D distribution can be expressed as $\varepsilon(\mathbf{r}) = \varepsilon_0 + \varepsilon_1(\mathbf{r})$. The total field is a sum of the unperturbed field of the source and the scattered component $\mathbf{E}(x,y,\omega) = \mathbf{E}_0(x,y,\omega) + \mathbf{E}_1(x,y,\omega)$. Variations of the received signal *s* are proportional to the convolution of the instrument function *F* of the receiver and the scattered field $\mathbf{E}_1(x,y,\omega)$. In frameworks of the Born approximation, for the proposed in [3-4] scheme of measurements with the fixed the source-receiver vector $\boldsymbol{\delta}\mathbf{r}$, when the structure of sounding field is invariable relative to the receiver position, it is possible to express the transversal spectrum (2D inverse Fourier transform over *x* and *y*) of measured signal variations as:

$$\begin{aligned} s(k_x,k_y,\omega) &= F_i(k_x,k_y,\omega)E_{1i}(k_x,k_y,\omega) = \\ &= \int_{z'} \varepsilon_1(k_x,k_y,z')\cdot \\ &\times i\pi\omega\, F_i(k_x,k_y,\omega)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(k_x+\kappa_x)\delta x+i(k_y+\kappa_y)\delta y} d\kappa_x d\kappa_y \cdot \\ &\times\{\int_{z''} [\, j_i(-k_x-\kappa_x,-k_y-\kappa_y,z''-z-\delta z)\cdot \\ &\times g_{ij}^{12}(k_x+\kappa_x,k_y+\kappa_y,z',z'')]g_{ji}^{21}(\kappa_x,\kappa_y,z,z')dz''\}dz', \end{aligned} \tag{1}$$

where g_{ji}^{lk} are *k*-space components of the Green tensor above and below the air-ground interface; j_i is the source current distribution. To obtain the transversal spectrum of a source, the surface current distribution on antennas has been calculated using CST Microwave Studio. We assume that the receiver has the same transfer function as the source ($F_i(k_x,k_y,\omega) = j_i(k_x,k_y,\omega)$). To solve the Fredholm integral equation (1) for each pair of spectral components, the algorithm based on the generalized discrepancy method in the complex Hilbert space W_2^1 has been worked out in [6] and applied here to solve the tomography problem for inhomogeneities with the complex-value distribution of permittivity. Finally, the desired 3D structure of permittivity is obtained by the 2D inverse Fourier transform:

$$\varepsilon_1(x,y,z) = \iint \varepsilon_1(\kappa_x,\kappa_y,z)e^{i\kappa_x x+i\kappa_y y}d\kappa_x d\kappa_y. \tag{2}$$

The depth sensitivity for multifrequency measurements is related to the frequency dependence both of the medium absorption and of the extinction depth of evanescent components of the probing field. For the targets in the near-field zone of electrically-small antennas, it is possible to achieve a subwavelength resolution of the proposed tomography.

3. RESULTS OF THE MICROWAVE TOMOGRAPHY

Applying the described approach for the experimental results [5], we have obtained tomography images for the studied inhomogeneities of the complex permittivity. Multifrequency measurements based on the 2D lateral scanning of the complex signal $s(x,y,f)$ for six chosen frequencies: 1.7, 2.76, 3.82, 4.88, 5.94, 7.0 GHz have been used in analysis. The source-receiver system include two identical bow-tie transmitting and receiving antennas with the length of arms 3.8 cm and the width of 5.4 cm, placed in the *y*-direction; the fixed distance between centers of antennas was δx =7.5 cm. They were scanning together in the rectangle *x*-*y* area above the range of the subsurface ice target with sizes 10×10×4 cm that has been buried in the sand at the depth *z* = –9 cm.

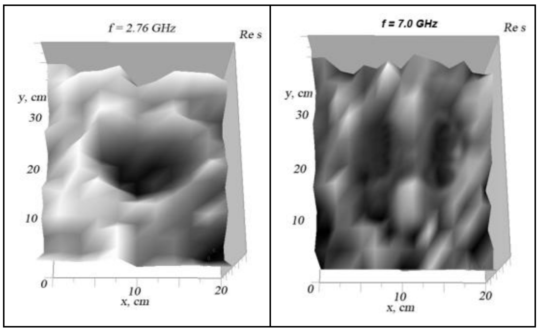

Fig. 1. Measured distributions of the received signal over the studied region at two frequencies. Left, $\mathrm{Re}\, s(x,y,f=2.76\,GHz)$; right, $\mathrm{Re}\, s(x,y,f=7.0\,GHz)$.

In Fig. 1 one can see the microwave image of the buried ice target at two frequencies f_2 = 2.76 GHz (wavelength 10.83 cm) and f_2 = 7.0 GHz (4.3 cm). This result shows the sensitivity of measurements to the subsurface inhomogeneous region. Also, it is possible to see that the relative level of measurement errors was high enough (about 0.2). But there is a possibility to reduce errors, involving in analysis data, averaged over frequency bands around reasonable chosen frequencies – using measurements at all available 800 frequencies. It makes possible to reduce the level of random errors by an order.

In this study, results of 2D scanning at 6 frequencies have been used to solve the integral equation (1) relative to the depth profile of the lateral spectrum of permittivity perturbation $\varepsilon_1(k_x, k_y, z)$ for each pair of spectral components. Then, from (2), we have obtained the desired 3D structure (tomogram) of the complex permittivity $\varepsilon_1(x,y,z)$ of the studied inhomogeneity. In Fig. 2 the vertical section of the retrieved 3D permittivity (tomogram) is presented; in Fig. 3 – the tomogram in the horizontal section at the depth of the ice target.

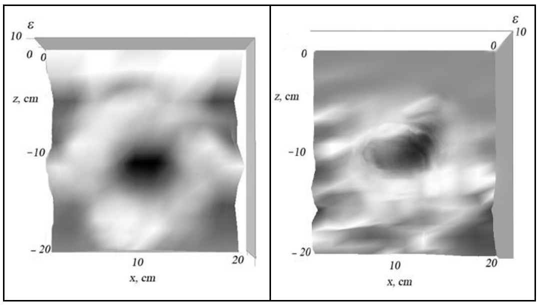

Fig. 2. Vertical section of the retrieved 3D permittivity (tomogram) at y = 16 cm. Left, $\mathrm{Re}\,\varepsilon_1(x,z)$; right, $\mathrm{Im}\,\varepsilon_1(x,z)$.

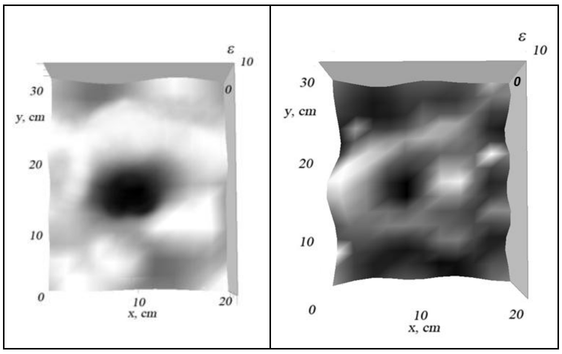

Fig. 3. Horizontal section of the retrieved 3D permittivity (tomogram) at z = – 9 cm. Left, $\mathrm{Re}\,\varepsilon_1(x,y)$; right, $\mathrm{Im}\,\varepsilon_1(x,y)$.

Results, taking into account a high enough level of errors, are in a reasonable correspondence with the expected distribution of the real and imaginary parts of permittivity, related to the buried melting ice. One can see regions of low values both for real and imaginary parts of permittivity approximately at the position of the ice target. There are also regions of enhanced values of these permittivity parameters around the ice location that could be expected because the influence of the enhanced water content related to diffusion of water because of melting.

4. CONCLUSION

We have applied a new method of near-field multifrequency coherent tomography in the microwave range to retrieve the subsurface 3D structure of complex permittivity. Our first results show the feasibility of this method for tomography of absorbing inhomogeneities. Further study should determine possible regions of this tomography application.

5. PERMISSION TO PUBLISH

The authors are responsible for all material contained in the manuscript they submit.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (grant *No.* 08-02-00117) and by the Program of the Russian Academy of Sciences.

REFERENCES

1. Gaikovich K. P. 2004, *Inverse Problems in Physical Diagnostics*, Nova Science Publishers Inc., New York.
2. Gaikovich K. P. 2007, ‘Subsurface near-field scanning tomography’, *Phys. Rev. Lett.* **98**, 183902)**;** *Phys. Rev. Lett.* **98**, 269902.
3. Gaikovich K. P. , Gaikovich P. K. 2008, ‘Scanning coherent tomography of multilayered media with inhomogeneous region’, 2008, *Proc. ICTON 2008, Greece, June 22-26*, Athens, 246-9.
4. Gaikovich P. K. 2008 ‘Scanning coherent tomography of multilayered media with inhomogeneous region’, *Proc. 4th International Conference “Ultrawideband and Ultrashort Impulse Signals” (15-19 September, Sevastopol, Ukraine),* Sevastopol, pp. 189-191.
5. Gaikovich K. P. 2010, ‘Inverse scattering problem in Hilbert space’ (invited), *Proc. ICTON 2010, 27 June – 1 July 2010,* Munich.
6. Gaikovich K. P., Gaikovich P. K., Maksimovitch Ye. S., Badeev V. A., Mikhnev V. A. ‘Near-field microwave tomography’, *Proc. of 7th International Conference on Antenna Theory and Technique (6-9 October, 2009 Lviv, Ukraine),* Lviv, 262-4.

Gaikovich K. P., Gaikovich P. K., Maksimovitch Ye. S., Badeev V. A. ‘Study of scanning microwave tomography of subsurface targets’, 2010, *Proc. of CriMico 2010, 13-17. Sept. 2010,* Sevastopol.