RETRIEVAL OF SURFACE THERMAL HISTORY FROM THE RADIO-THERMAL SPECTRUM

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UDC 621.371.551.526

The authors have obtained thermal evolution equations directly connecting the observed thermal emission spectrum of a half-spare with the evolution of its' surface temperature (or the heat flux through the surface) prior to the measurement. From measured data on the thermal radio-emission of soil at wavelengths 0.8, 3, 9, and 13 cm, using the relations obtained, the thermal history of the ground surface has been obtained, and it has been shown that one can evaluate this parameters remotely (dielectric permittivity, thermal diffusivity, humidity). This method is proposed for retrieving the thermal history of glacial land masses in the remote past and for evaluating the thermal and dielectric parameters of planet surfaces.

To retrieve the thermal history from the thermal emission spectrum of a medium is of interest for a wide class of subjects. In this paper we consider the case of a half-space with a uniform temperature distribution (uniform with depth) for which we can obtain closed-form equations and apply a clear interpretation. This case can have a practical application in investigating the thermal dynamics of the Earth's surface and the surface of other planets from the measured thermal emission in the radio range. The limiting probe depth (and correspondingly, the time scale for the surface thermal history) is determined by the thickness of the skin-layer, which is proportional, as a rule, to the wavelength, and in the decimeter range can be several, centimeters for moist soil, several meters for dry soil, and several hundreds of meters in the glacial lands of the Antarctic. Correspondingly, the time scales of variation of surface temperature, which determine the variation of the temperature profile at the above values of depth, vary from several hours to several hundred years. Thus, with this approach we can investigate both the. processes of the daily thermal soil dynamics, and the climatic variations on the surface in the past. The equation connecting the brightness. temperatures of the half-space with the evolution of its surface temperature contains as parameters the absorption coefficient and the thermal diffusivity. By measuring brightness temperature in the microwave range, and the evolution of temperature

Institute for Scientific Research in Radiophvsics. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 32, No. 11, pp. 1343-1350, November, 1989. Original article submitted May 4, 1988.

in the IR range, we can use the relations obtained here to determine these parameters from remote measurements, and from these, in turn, to determine the physical characteristics of the soil (e.g., humidity and density) and draw conclusions regarding its morphology.

l. Interaction of the Radiative Spectrum of a Half-Space with Evolution of the Boundary Conditions. We consider the half-space z < 0 with complex dielectric permeability $\varepsilon = \varepsilon' - \varepsilon''$. THE brightness temperatures at wavelength λ for the case of measurements at the nadir are connected in the radio range with the temperature profile through the relation

$$T_B(\lambda) = (1-R(\lambda)) \int_{-\infty}^{0} T(z)\gamma(\lambda, z) \exp\left[-\int_{z}^{0} \gamma(\lambda, z')dz'\right]dz, \qquad (1)$$

where $R(\lambda)$ is the Fresnel reflection coefficient, and $\gamma = (4\pi/\lambda)\,\mathrm{Im}(\sqrt{\varepsilon})$. In the case of a uniformly moistened medium the value of γ does not depend on *z,* as a rule [1, 2]. If the boundary temperature is a function of only the time, $T_0(t)$, the unique solution of the heat conductance equation

$$\frac{\partial T}{\partial t}(z, t) = a^2 \frac{\partial^2 T}{\partial z^2}(z, t) \qquad (2)$$

in the region ($t > -\infty$, $z < 0$), satisfying the boundary condition $T_0(0, t) = T_0(t)$, has the form (see, e.g., [3])

$$T(z, t) = \int_{-\infty}^{t} -\frac{zT_0(\tau)}{\sqrt{4\pi a^2(t-\tau)^3}} \exp\left[-\frac{z^2}{4a^2(t-\tau)}\right]d\tau. \qquad (3)$$

As *t* increases the contribution of the special features of $T(\tau)$ for remote times decreases. On the other hand, for any *z* there is a time interval adjoining time *t* for which the contribution to the integral of values $T_0(\tau)$ is also very small (the perturbation does not reach the corresponding depth), i.e., for any depth there is a time in the past for which the contribution in Eq. (3) is a maximum.

The distribution $T(z)$ at time *t* can also be determined if we know the heat flux through the surface as a function of time. The heat flux is $J_0 - k(dT/dz)(z = 0)$ (*k* is the thermal diffusivity). The corresponding boundary problem for Eq. (2) for the condition $dT/dz(0, t) = -(1/k)J_0(t)$ has the solution [3]

$$T(z, t) = \int_{-\infty}^{t} -\frac{J_0(\tau)}{k\sqrt{\pi(t-\tau)}} \exp\left(-\frac{z^2}{4a^2(t-\tau)}\right)d\tau. \qquad (4)$$

From Eqs. (3) or (4) we can obtain equations which directly connect the brightness temperatures of the thermal radio-emission of a half-space, measured at time *t*, with the evolution of its surface boundary conditions. We substitute Eqs. (3) and (4) into Eq. (1) and change the limits of integration with respect to *z* and *t*. As a result we have

$$T_B(\lambda) = (1-R)\int^{t} T_0(\tau)K(\tau, \lambda)d\tau, \qquad (5)$$

where for γ = const

$$K(\tau, \lambda) = \int_{-\infty}^{0} -\frac{\gamma z}{\sqrt{4\pi a^2(t-\tau)^3}} \exp\left[\gamma z - \frac{z^2}{4a^2(t-\tau)}\right]dz.$$

Analogous]y,

$$T_B(\lambda) = (1-R)\int_{-\infty}^{t} J_0(\tau)\tilde{K}(\tau, \lambda)d\tau,$$

$$\tilde{K}(\tau,\lambda) = \int_{-\infty}^{0} -\frac{a\gamma}{k\sqrt{\pi(t-\tau)}} \exp\left[\gamma z - \frac{z^2}{4a^2(t-\tau)}\right] dz . \quad (6)$$

The generalization of Eqs. (5) and (6) for the case $\gamma=\gamma(z)$ is trivial. However, for non-uniform γ we must also consider non-uniform specific heat and thermal conductivity, and this problem has not been solved in closed form, analogous to Eq. (3). We note that [2] proposed different approaches to solving the heat conduction equation, allowing for non-uniform characteristics, but in the present paper we restrict our attention to the case when a^2 and γ are independent of depth.

From Eqs. (5) and (6) one can formulate the inverse problems from the measured $T_B(\lambda)$ at time t to retrieve the surface thermal history (i.e., evolution of temperature and heat flux). We shall call Eqs. (5) and (6), respectively, the first and second thermal evolution equations for brightness temperatures.

It should be noted that the inverse thermal problems of retrieving the thermal history from direct measurements of $T(z)$ on the basis of the integral equation (3) are known, and have been examined, e.g., in [2,4] . However, direct measurements are often difficult to accomplish and may require the structure of the medium to be destroyed. This problem can be solved if we use the $T(z)$ profile, retrieved from the data of remote radiometric measurements $T_B(\lambda)$ from Eq. (1). However, this approach requires sequential solution of two incorrectly posed problems for a Fredholm equation of the first kind, which is difficult to accomplish with acceptable accuracy. The error of the retrieved profile T(z) is several times greater than the error of measurement of T_B. The advantage of this approach is that Eqs. (5) and (6) give a direct connection between the spectrum T_B and the surface thermal history, from which one can solve the inverse problem of retrieving $T_0(t)$ or $J_0(t)$, respectively, on the basis of only one equation.

It follows from Eq. (3) that the maximum contribution to the temperature at depth z at time t is made by the temperatures T(τ_m) at time τ_m, which precedes the time t by the amount

$$t - \tau_m = z^2/6a^2. \quad (7)$$

Thus, the scales of time end depth of probing are connected. The more transparent is the medium to the radiation, the more do the deep layers contribute to T_B. The temperature of these layers is formed under the influence of previous surface conditions. From Eqs. (5) and (6) we can retrieve these conditions, i.e., the evolution of $T_0(t)$ and $J_0(t)$.

The most important application of Eqs. (5) and (6) may be their use to retrieve the evolution of $T_0(t)$ and $J_0(t)$ from the measured time dependence $T_B(t)$ at one wavelength, i.e., to accomplish monitoring of the surface temperature and heat flux. Here Eqs. (5) and (6)' should be regarded as a Volterra equation of the first kind with a variable upper limit t. And thereby, taking account of Eqs. (3) , we also solve the problem of one-wavelength probing of the dynamics of the surface profile $T(z,t)$ and the solution is practically correctly posed, as is known for equations of Volterra type.

The thermal evolution equations obtained can also be used for remote determination of soil parameters (dielectric permittivity, thermal diffusivity). By treasuring the spectrum of T_B and also the surface temperature dynamics (which can also be done, remotely with an IR radiometer), we can use Eq. (5) and the least squares method, for example, to evaluate the parameters entering into Eq. (5). The accuracy of the evaluation depends in a given case also on the form of $T_0(t)$ (for $T_0(t)$ = const, $T_B=T_0$ independently of the values of the ground parameters).

2. Solution of the Integral Equation. The problems of retrieving $T_0(t)$ from Eq. (5) or J_0 from Eq . (6) from the $T_B(\lambda)$ spectrum represent integral equations of Fredholm type of the first kind, and are incorrectly posed in the sense or [4]. To solve them one must enlist enough *a priori* information about the properties of the exact solution. We used the Tikhonov regularization method in the form of the generalized discrepancy principle [5]. We rewrite Eq. (5) in the compact operator form

$$AT_0 = T_B^{\delta} \quad (8)$$

where T_B^{δ} is the measured realization of the right side, for which the measure of accuracy δ is estimated in the metric L_2 (see [5, 6]. To find the approximate solution $T_0^{\delta}(t)$ of

Eq. (5) in the set of differentiable functions we minimize the functional

$$M^{\alpha}(T_0) = \| A T_0 - T_B^{\delta} \| + \alpha \left(\| T_0 \|^2 + \left\| \frac{d T_0}{dt} \right\|^2 \right) . \tag{9}$$

where $\| x \|$ denotes the norm in space L_2. The regularization parameter α is determined from the equation of generalized discrepancy

$$\rho(\alpha) = \| A T_0^{\delta} - T_B^{\delta} \| - \delta^2 = 0 , \tag{10}$$

where T_0^{δ} is the function minimizing Eq . (9). The additional *a priori* information, e.g., that the exact solution is nonnegative is taken into account in the minimization of Eq. (11), allowing for the appropriate limits.

It should be stressed that in solving the incorrectly posed problems examined one cannot set up relations that are valid in all situations between the error of measurements and the error of retrieval. This relation depends appreciably on the form of the kernel, on the class of retrieved distributions, and also on the nature of the distribution and the values of the errors themselves. With a reduction of measurement error the accuracy of the solution increases, but not proportionally, as in correctly posed problems, but much more slowly. For a given measurement accuracy only a limited set of wavelengths is informative, since the variations of T_B are close to the value at the closest wavelengths, because of the smoothing action of the kernel. The difference of the variations depends both on the kernel of the equation, and on the form of the retrieved function. For a given type of distributions the discretization of the measurements with respect to λ is determined by the accuracy or the measurements of δT_B because of the natural condition for the difference of the variations at the adjoining wavelengths $\Delta T_B(\Delta\lambda) \geq \delta T_B$.

It is clear from what has been said that questions relating to the accuracy requirements of the measurements, the set of wavelengths, and estimates of the retrieval error can be solved only by a numerical experiment in a closed scheme (brightness temperatures were computed from model distributions, the error with a given distribution was drafted out with the aid of a random number sensor; with the "data" thus obtained we solved the inverse problem and the results of retrieval were compared with the original distribution). Taking into account that we have used the Tikhonov method in a number of papers [6-10] to solve similar equations, where the results of numerical modeling were also given, we shall restrict ourselves to giving some general conclusions and specific results for the conditions of the experiment .

When the kernel of Eq. (8) is smooth the quality of the retrieval depends decisively on the form of the retrieval function. Functions with one maximum or mono tonic function that are uniformly distributed over the integration interval are retrieved at an error level of 10% from measurements at 3 or 4 wavelengths with accuracy 20-30%. To achieve comparable accuracy in retrieving functions with two maxima the requirements as to measurement accuracy increase by an order of magnitude, and the required number of wavelengths is not less than 10 (see an analogous example in [6]).

The accuracy of the solution increases if we seek a solution in the form of a deviation from a chosen "average" or "probable" distribution $T_0°(t)$ from physical considerations. Here the deviation has a simpler form than the function itself. It can be chosen equal to zero at the top end of the interval, since the surface value T_0 is known, which allows us to identify the part of the integral giving information lo the more remote times. In the case where the problem is solved for the variations, the estimates made above, also refer to the corresponding deviations; i.e., to retrieve the variations T_0 (t) of amplitude 5-10 K with accuracy 1-2 K one must measure with accuracy 0.2-0.3 K. With such measurements it is appropriate to form a set of 3 and 4 wavelengths in such a way that, the time intervals computed by substituting the skin layer depth $d_c = 1/\gamma$ it the chosen wavelengths into Eq. (7) will span, as uniformly as possible, (the information range of retrieval determined by the maximum wavelength. This condition is not very critical with respect to small variations in the choice of wavelength.

3. Determination of Parameters in Retrieving the Daily Dynamics of Ground Surface Temperature from the Spectrum of its Brightness Temperatures. In October of 1986 in the Karadag polygon NIRFI (southeast bank of the Crimea) three days of continuous measurements

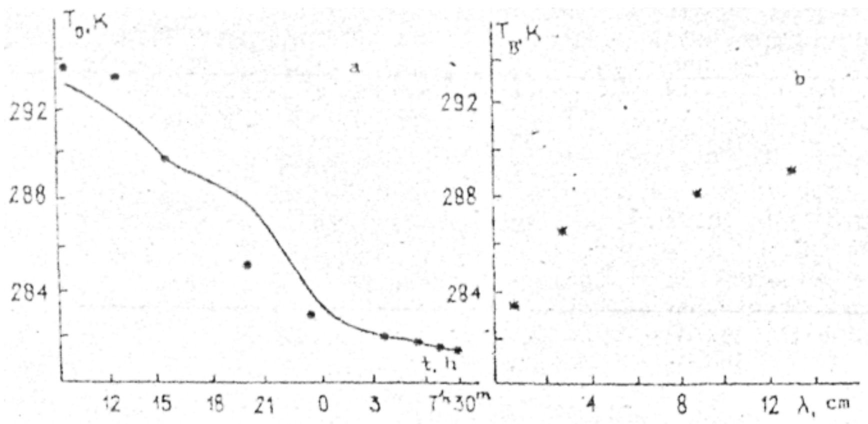


Fig. 1. Retrieval of the evolution of the soil surface temperature $T_0(t)$ from measurements of $T_B(\lambda)$ at 7:30 on 10-6-86; a) the solid curve is the retrieved $T_0(t)$; the points are the contact, measurement data; b) the measured values of $T_B(\lambda)$.

were made of soil brightness temperatures at wavelengths 0.8, 3, 9, and 13 cm, along with simultaneous contact measurements of its surface temperature $T_0(t)$. In the measurements a special technique was used to eliminate the influence on the brightness temperature of reflection and scattering of the ground by locating the receiving antenna under a reflecting screen, described in detail in [6, 7]. Thereby we eliminated the uncertainty dim to the difficulty to control influence of the above factors on the form of the dependence of brightness , and simplified the calibration of the measuring system. With this method we could achieve the accuracy of measurement $\delta T_B \sim 0.1$-0.3 K necessary to solve inverse problems. Allowing for the screening, Eqs. (1) and (5) acquire the form (γ is independent of depth)

$$T_B(\lambda) = \int_{-\infty}^{0} T(z)\gamma(\lambda)e^{\gamma z}dz; \quad (11)$$

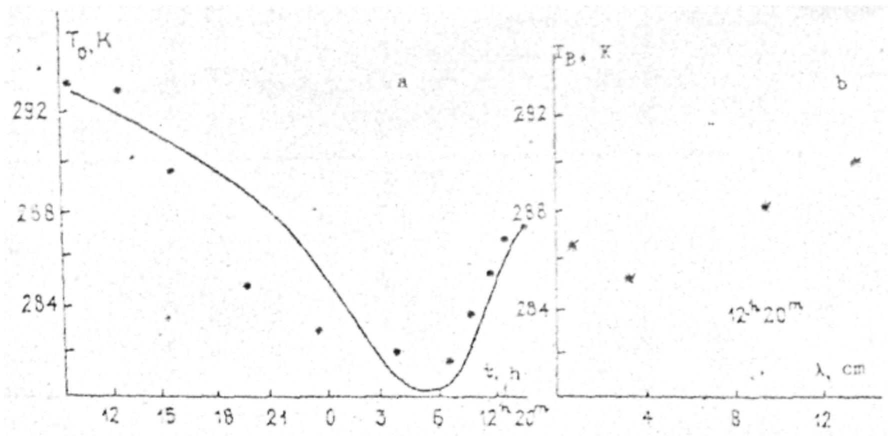
$$\dot{T}_B(\lambda) = \int_{-\infty}^{t} T_0(\tau)K(\tau,\lambda)d\tau. \quad (12)$$

As has been noted, the thermal evolution equation (12) can be used in two ways. For example, if we know the measured spectrum T_B the evolution of the surface temperature $T_0(t)$, we can solve the problem of determining the parameters γ and a^2 on which the kernel K depends [see Eq. (5)]. On the other hand, it we know the form of the kernel $K(\tau, \lambda)$, we can use Eq. (12) to solve the problem of retrieving the surface history $T_0(t)$ from the spectrum $T_B(\lambda)$. To investigate the achievable accuracies in solving each of the problems formulated we used the data of the experiments performed.

To solve the problem of determining the soil parameters is simplified as follows. It is known that for a given soil morphology, the absorption coefficient, the dielectric parameters, and the thermal diffusivity, which determine the form of the kernel $K(\tau,\lambda)$ in Eq. (12), depend only on the soil humidity *m.* The appropriate relations were given in [2, 11]. Therefore, from this we can evaluate the basic soil parameters from a single measurement of T_B at one wavelength, although more accurate results obtained by applying the method of least squares to all the measurements at the different frequencies. One can regard the thermal diffusivity as the independent parameter, and obtain the parameters *m* and a^2 from the codition

$$\sum_i \left\{ \dot{T}_B(\lambda_i) - \int_{-\infty}^{t} T_0(\tau)K[\gamma(m), a^2, \tau]d\tau \right\}^2 = \min. \quad (13)$$

The dependence $\gamma(m)$ is known [11]. From the approach described the estimate of soil humidity $m = 10$ % obtained. The value of *m* determined by the thermostat-gravimetric method was 14±2%.



FJR. 2. Retrieval of $T_0(t)$ from the measured TB(λ) at 12:20 on 10-6-F6. The symbols are analogous co those in Fig. 1.

The method described may be of interest for remote determination of the parameters of the soil surface layer of other planets and asteroids. As a rule, the soils of these heavenly bodies do not have water in the free state, and therefore they have, the properties of dielectric without dispersion, for which R, $\gamma\lambda$ = const, and the kernel of Eq.(12) $K = K(a^2, \gamma\lambda)$ depends on two parameters. These parameters, as well as the reflection coefficient R, can be determined from complex microwave and IR, measurements from the condition

$$\sum_i \left\{ T_B^{SHF}(\lambda_i, t) - (1-R)\int_{-\infty}^{t} T_B^{IR}(\tau) K(a^2, \gamma\lambda, \tau)\, d\tau \right\}^2 = \min .$$

From the values of R and γ we determine the dielectric parameters ε' and ε'' of the medium, and from ε' we can evaluate the soil density [12].

We now consider the problem of retrieving the thermal history $T_0(t)$ by solving Eq. (12) with the Tikhonov method. During the experiment the soil was uniformly moistened, which allowed the use of expressions for uniform hall-space. The skin layer thickness at the wavelength used spanned, a layer, roughly uniformly, of depth on the order of 15 cm. According to estimates using Eq. (7), the main temperature contractions associated with the daily dynamics of the soil surface temperature, span a layer of soil of thickness 15-20 cm. Since the probed depth spans this layer, all the prerequisites are met for solving the problem of retrieving the daily dynamics of the soil surface measured $T_B(\lambda)$ data. A comparison of retrieved results with data of direct contact measurements of $T_0(t)$ show that the problem can be solved accurately.

Figures 1 and 2 show the results of retrieving $T_0(t)$ by solving Eq.(12) with respect to the measured $T_B(\lambda)$ at times 7:30 (Fig. 1.) and 12:20 (Fig.2) on 10-6-87. Figure 1shows the retrieval of monotonic nocturnal cooling of the soil, and Fig. 2 shows non-monotonic temperature evolution of the surface, including a period of warming after sunrise, and a period of nocturnal cooling after sunset. In the retrieval by the Tikhonov method the limit $T_0(t) \leq 20^0$C was used.

Apropos the possible applications of the method of prospective retrieval of the thermal history on the surface of icy continents at a distant epoch, it should be stressed that this problem is completely equivalent to that considered above, of probing soil, from the viewpoint of the possibilities of the method in principle, and is different only in regard to the scales of depth and time. It is known that heat conduction problems are equivalent, if when the depth scale is changed by a factor of n, we change the time scale by a factor of n^2 i.e., $T(z, t) = T(nz, nt^2)$ (see, e.g., [3]).

The methods developed can be used in planetary radio astronomy, and also for objects that differ in nature, shape, and spectral range.

The authors thank D.M. Gordeev for his help in the measurements.

LITERATURE CITED

1. A. M. Shutko, Microwave Radiometry of an Aqueous Surface and Ground Soil [in Russian], Nauka. Moscow (1986) .
2. A. F. Chuddnovskii, Thermophysics of Soils [in Russian], Nauka, Moscow (1976).
3. A. N. Tikhonov and A. A. Samarskii , Equations of Mathematical Physics [in Russian), Nauka, Moscow (1977).
4. A. N. Tikhonov and V. Ya . Arsenin, Methods of Solving Incorrectly Posed Problems [in Russian], Nauka, Moscow (1979).
5. A. N. Tikhonov, A. V. Goncharskii, V. V. Stepariov, and A. C. Yagola, Regularizing Algorithms and *a priori* information [in Russian], Nauka, Moscow (1983).
6. K. P. Gaikovich, A. N. Reznik, M. I. Sumin, and K. V, Troitskii, Preprint of NIRFI No. 214. [in Russian], Gor'kii (1986).
7. K. P. Gaikovich, A. N. Reznik, M. I. Sumin, and R. V. Troitskii, Izv. Akad. Nauk SSSR, Ser. FAO, 23, No. 7, 761 (1987).
8. N. A. Vasilenko, K. P. Gaikovich, and M. I. Sumin, Dokl. Akad. Nauk SSSR, 290, No. 6, 1332 (1986).
9. K. P. Gaikovich and M. I. Sumin, Reports of the 7th All-Union Conf. on Radio-Meteorology [in Russian], Moscow (1986), p. 6.
10. K. P. Gaikovich, M. I. Sumin, and R. V. Troitskii, Radiofizika, 31., No. 9, 1104 (1988).
11. M. T. Hallikainen, F. T. Ulaby. M. A. El-Rayes, and L. Wu, IEEE Trans. Geosci. Remote Sensing, Vol. 23, No. 1, 25 (1985).
12. V. D. Krotikov, Izv. Vyssh. Uchebn. Zaved.. Radiofiz., 5, No. 6, 1057 (1962).