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GROUND-BASED DOPPLER RADIO-FREQUENCY REFRACTOMETRY OF THE ATMOSPHERE

K. P. Gaikovich

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An *exact relationship between the Doppler shift of a satellite signal and astronomical refraction is used to reduce the problem of reconstruction of the altitude profile of the refractive index, to an inverse refraction problem. The range of observation angles in which measurements are informative is determined. Mathematical modeling is used to obtain statistical estimates of the possible reconstruction accuracy for various sets of climatic conditions, and the required accuracy for measurement of Doppler frequency and satellite orbit parameters is determined.*

Reconstruction of the altitude profile of the refractive index *(n)* according to the atmosphere's contribution to the Doppler shift of the signal from a space vehicle has been used widely in radio-frequency transillumination of the planets of The solar system (see Kliore and Lions el al. [1, 2], for example). In this situation, when the source and receiver are located outside of the atmosphere being studied, the problem is solved by applying an inverse Abelian transformation to the observed dependence of the measured quantity on the target parameter. The problem of refraction for sources in the optical region is solved similarly [3, 4J.

For the measurement geometry shown in Fig. 1, when a satellite signal is received on the earth's surface, solution of the problem is greatly complicated. The problem was formulated in this way for the first time by Kolosov and Pavel'ev [5J and solved by Armand et al. [6]. Here, we were able to overcome a number of difficulties and avoid approximations that prevented Kolosov and Pavel'ev from realizing the full potential of the method. The principal procedural aspects of the solution of this incorrect problem for the case of optical observations were examined earlier [7, 9), but the additional contribution of humidity to the refractive index must be taken into account in the examination of radio-frequency refraction, which requires separate analysis.

I. CALCULATION OF DIFFRACTION FROM DOPPLER SHIFT

A relationship between the Doppler frequency ν_D and the parameters of the atmosphere is obtained by differentiation of the expression for the signal phase and has the form [5|

$$\nu_D = \frac{\nu}{c r_H} \left(p_0 \cos\theta_0 V_\perp + \sqrt{r_H^2 - p_0^2 \cos^2\theta_0}\, V_R\right), \quad p_0 = n_0 r_0, \tag{1}$$

where r_H, $V_\perp$, and V_R are the geocentric distance and velocity components of the source; ν is frequency; θ_0 is elevation; and $n_0 = n(r_0)$ is the refractjve index at the surface.

It is apparent from (I) that if the geometrical location of the satellite and its velocity vector are known, the elevation angle of ray arrival θ_0 can be determined by measuring ν_D on the earth's surface and solving a simple equation.

Then, the refraction angle of an equivalent infinitely distant (along the ray) source is determined through the coordinates (central angle a and geocentric distance) of a real source

$$\varepsilon = \alpha - \arccos \frac{p_0 \cos\theta_0}{r_H} + \theta_0, \tag{2}$$

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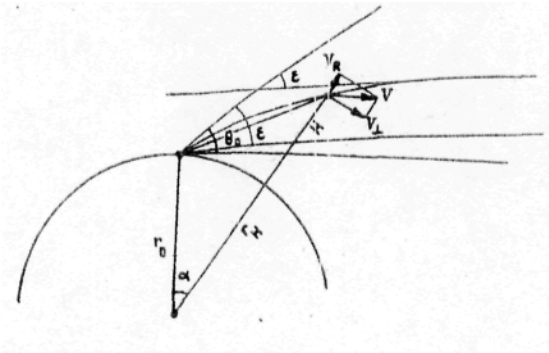

Fig. 1

TABLE 1. Refraction Characteristics for Oceanic Ensemble

	θ_o							
	0.01°	0.1°	0.5°	1°	2°	3°	4°	5°
$\langle\varepsilon\rangle$	0.94°	0.9°	0.74°	0. 60° "	0.43°	0.33°	0.26°	0.22°
σ_ε	6'58"	5'18"	2 '40"	1'51"	1'12"	53"	41"	34"
σ_ε/N_0	6 '05"	4 '22"	1'30"	39"	12"	4.7"	2.3"	1.2"

for which

$$\varepsilon(p_\theta) = -p_\theta \int_{p_0}^{\infty} \frac{d \ln n}{dp} \frac{dp}{\sqrt{p^2 - p_\theta^2}}, \quad p = nr, \quad p_\theta = n_0 r_0 \cos\theta_0. \tag{3}$$

Or, integrating (3) by parts, as earlier [7-9]:

$$\int_{p_0}^{\infty} N(p) \frac{p p_\theta}{(p^2 - p_\theta^2)^{3/2}} = \tilde{\varepsilon}(p_\theta), \tag{4}$$

where

$$\tilde{\varepsilon}(p_\theta) = -10^{-6}\varepsilon(p_\theta) + p_\theta \frac{N(p_0)}{\sqrt{p_0^2 - p_\theta^2}}. \tag{5}$$

Equation (4) is a linear Fredholm's integral equation of the first kind with respect to the distribution $N(p)$, which is converted to an altitude profile $N(h)$ by calculation of the altitude scale:

$$h = \frac{p}{n} - r_0 = \frac{p}{1 + 10^{-6}N(p)} - r_0. \tag{6}$$

2. DETERMINATION OF USEFUL MEASUREMENT-ANGLE RANGE

As is known [10], the refractive-index distribution shows appreciable refraction-angle variations only at fairly small elevations θ_0. With an increase in the angle, the variations are reduced and refraction is determined with increasing accuracy by the value of the refractive index at the earth's surface, which is the substance of the so-called "Laplace theorem." Thus, the range

TABLE 2. Refraction Characteristics for Ensemble Uncles Summer Conditions in the European Territory of the USSR

	θo							
	0.01°	0.1°	0,5°	1°	2°	3°	4°	5°
$\langle\varepsilon\rangle$	0,70°	0,67°	0,58°	0,48°	0,35°	0,27°	0,22°	0,18°
σ_ε	3'26"	3'16	2'35"	1'55"	1'13"	52"	40"	33"
σ_ε/N_0	1'30"	1'17"	41"	21"	7″	3,2"	1,7"	1,0"

of angles that are useful for solution of the inverse problem is limited from above by the angles at which the natural variations of refraction become comparable with the measurement error.

Table 1 lists the results of calculations of the climatic-mean values of radio-frequency refraction (e) and its variations σ_ε (standard deviation from $\langle\varepsilon\rangle$) and σ_ε/N_0, which were determined as the standard deviation of the refraction as calculated from aerological data ("true" refraction) from the refraction calculated from the profile $N^e(h)$ obtained by extrapolation from the surface value of the refractive index N_0 as

$$N^e(h) = \langle N(h)\rangle + \frac{B_{NN}(0, h)}{\sigma_N^2(0, 0)} (N_0 - \langle N_0\rangle), \qquad (7)$$

where B_{NN} is a covariance matrix and σ^2_N is the variance. The data in Table I correspond to a ensemble of data for a tropical ocean (80 realizations). Similar parameters for a summer ensemble for the European territory of the USSR are provided in Table 2. It can be seen that the mean values and variations of refraction are greater for the oceanic ensemble than for the continental conditions, which is a result of the high moisture content above the ocean. It is clear that to obtain data on the profile $N(h)$ with an accuracy that exceeds that of the statistical estimate for the surface value (7), the measurement error must be smaller than σ_ε/N_0. The range of useful angles $\theta_0 < 5°$ for a measurement error $\delta\varepsilon$= 10", which can be considered limiting for refraction measurements even in the optical region, and $\theta_0 <$ 2-3° for $\delta\varepsilon$ = 10".

Thus, the constraints on the range of useful angles are very substantial in the problem in question. The limits of the useful range at radio frequencies are close to those determined earlier [8] for the optical region.

3. METHODS FOR SOLUTION OF INVERSE PROBLEM

a) Solution Method for Compact Set of Monotonic Functions

The method employs an algorithm described earlier [7-9], when it was used to solve the inverse refraction problem in the optical region. The method consists essentially of minimization of the residual on a compact set of monotonic functions that are bounded from above and below by constants [1]. Mathematical modeling by a method described earlier [8j showed that the accuracy of the method was greatly dependent on the choice of a first approximation. The best results are obtained when extrapolated profile (7) is used as a first approximation. The fact that the iteration process can converge on a discontinuous function is unfavorable from the point of view of reconstruction quality. Satisfactory results (better than statistical) are obtained when $\delta\varepsilon \leq 1"$.

b) A. N. Tikhonov's Method of the Generalized Residual

The method employs *a priori* information on the smoothness of the exact solution jllj and allow» the introduction of constraints (the exact solution is greater than or less than a specified function). The method has been used successfully to solve various problems [12-15], including the problem in question [6|. In the case of Armand et al. [6], however, for the reasons mentioned above, the full potential of the method was not realized, and, what is more, it was concluded on the basis of a numerical experiment that there was a maximum attainable measurement accuracy, which is in contradiction with theory [11], in which conversion on an exact solution when $\delta\varepsilon\to 0$ was demonstrated, just as with the results of our numerical experiments. It

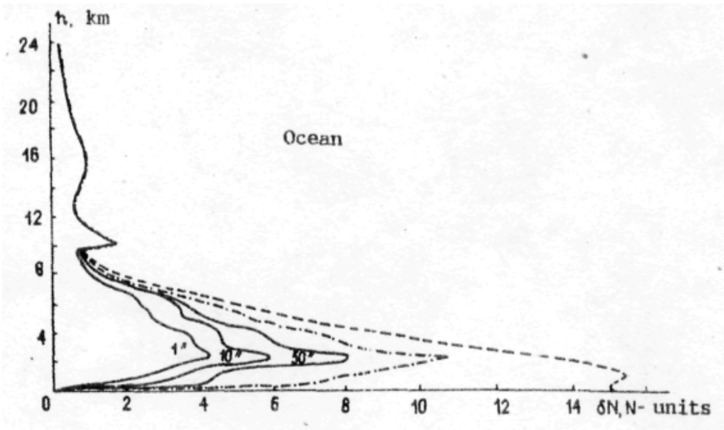

Fig. 2

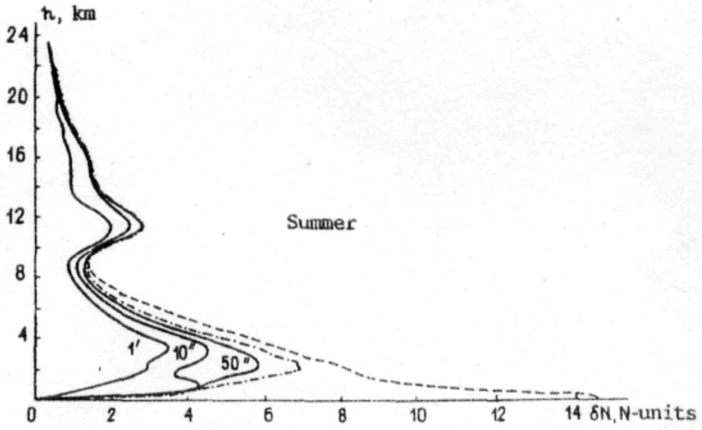

Fig. 3

is possible that such a result was obtained because with the reduction of error, there was not a simultaneous increase in the quantization of the integral or in the quantization of the angular dependence of the right side.

The numerical experiments that were performed showed that up to error level $\delta\varepsilon$= 10", the reconstructed profiles were, as a rule, closer to the starting profiles than were the statistically extrapolated profiles (the solution could be sought as the deviation from $N^e(h)$).

c) Method of Statistical Regularization

The most-accurate results were obtained by the method of statistical regularization [16], which we also used successfully earlier [7-9J. The method uses directly information on the interlevel covariance relations of $N(h)$. The solution is sought in an *a priori* ensemble that is specified by a covariance matrix. The probability density for an *a priori* ensemble is determined horn the condition of maximum entropy, and the *a posteriori* distribution is found by the Bayes formula.

The average vector N over this distribution for a given right side of (5) determines a solution that satisfies the equation

$$N = \langle N \rangle + (K^* W^{-1} K + B_{NN}^{-1}) K^* W^{-1} (\tilde{\varepsilon} - \langle \tilde{\varepsilon} \rangle), \qquad (8)$$

where K is the matrix of the kernel in (5) and W is the covariance matrix of the measurement errors.

The mean-square error *6N* of refractive-index reconstruction for the oceanic ensemble for various simulated measurement errors (1", 10", and 50") with variance σ_ε^2 is compared in Fig. 2 with the error of optimal extrapolation in N_0 (dot—dash curve) and the natural variation σ_N (dashed curve). The results of a similar statistical analysis for the summer ensemble of the European territory of the USSR are provided in Fig. 3. It can be seen that the refraction measurements are useful in each case,

TABLE 3

$\delta\varepsilon$	$\delta\nu_D$, Hz	δH, *m*	δV, cm/ sec	δN_0, *N*-units	δh_0, m
1"	10^{-3}	0.7	0.1	0.1	0.6
10"	10^{-2}	7.0	1.0	1.0	6.0
50"	$5\cdot10^{-2}$	35.0	5.0	5.0	30.0

i.e., solution of the inverse problem appreciably improves the results of optimal extrapolation at a measurement-error level $\delta\varepsilon \lesssim 50''$. The mean-square reconstruction error is highly inhomogeneous with respect to altitude and has two maxima, the lower of which apparently corresponds to the altitudes at which clouds are formed while the upper corresponds to the altitude of the tropopause. The reconstruction 31x1(13(7 rises not in proportion to the measurement accuracy but considerably more-slowly, which is typical feature of incorrect problems. Reconstruction is most-effective at altitudes of up to 4-6 km.

4. REQUIREMENTS ON DOPPLER MEASUREMENTS

As is apparent from (2), the accuracy of e determination is a function of the accuracy of determination of the angle 0_0 from ν_D measurements on the basis of relation (1). In addition to the Doppler frequency, relation (1) includes the altitude and velocity components V_R and $V_\perp$ of the satellite as well as $p_0 = n(r_0)r_0$ i.e., the altitude of the observation point h_0 and the value of (the refractive index at the earth's surface N_0 must also be known. Table 3 lists the errors of these parameters, which result in a corresponding value of error $\delta\varepsilon$ for use of the high-frequency channel of the "Tranzit" navigation system (ν = 400 MHz and H = 1000 km).

The high requirements on the accuracy of measurement of Doppler frequency ν_D that follow from the data in Table 3 arc a result of the relatively small contribution of the atmosphere to the Doppler shift. Calculations have shown that for the case in question this contribution is 3-4 Hz at the horizon and decreases by an order of magnitude at ($\theta_0 \approx 5°$, while ν_D is ~9000 Hz. It follows from Table 3 that it is very difficult to achieve an error of refraction measurement of better than 10"; the curves in Figs. *2* and 3 that correspond to this error level characterize the maximum attainable accuracies of Doppler radio-frequency refractometry. The data in Table 3 also show that in interpretation *of* the measurements it is necessary to take into account the additional frequency shift caused by the relativistic transverse Doppler effect $\Delta\nu_\perp = -\nu(V!c)^2 \approx -0.14$ Hz, whose magnitude is comparable with or exceeds the permissible measurement errors.

CONCLUSION

The possibilities of determination of the altitude profile of the refractive index for radio waves in the atmosphere from measurements of the Doppler shift of a satellite signal were examined theoretically. The exact relationship between the Doppler frequency and refraction in the atmosphere established by Kolosov and Pavel'ev [5] was employed, which allowed the reconstruction problem to be reduced to an inverse refraction problem. Statistical analysis of the refraction variations showed that measurements were useful in the range of small satellite elevation angles. Reconstruction of the profile $N(h)$ by various methods was mathematically modeled. The best results were obtained by the method of statistical regularization. It was established that the minimum attainable error of refraction determination was $\delta\varepsilon \approx 10''$ for measurements of the Doppler signals of the signals emitted by the satellites of the "Tranzit" navigation system. This requires measurements of ν_D with an error of not more than 0.01 Hz, of satellite altitude with an error of ≤7 m, and of satellite velocity with an error of ≤ 1 cm/sec. The range of satellite elevation angles in which measurements arc useful is $\theta_0 < 2\text{-}3^0$. *A* statistical analysis of the accuracy of reconstruction *of* $N(h)$ as a function of altitude was performed. For $\delta\varepsilon = 10''$, the mean accuracy of reconstruction in a layer of 1-5 km is better than 4 *N*-units, i.e., the method allows natural variations of the profile $N(h)$ to he traced with an accuracy of 25-50%.

The obtained estimates, while showing definite prospects for radio-frequency refractometry, undoubtedly require experimental verification, since it is difficult to evaluate within the framework of theoretical analysis the effect of horizontal inhomogeneities that have been insufficiently studied under various conditions. The "Tranzit" satellite navigation system, which allows data to be obtained practically every hour, could be used for an experimental treatment of the method.

The outlook for development of the method involves studies of the possibilities of reconstruction in measurements from a certain altitude at negative elevation angles, since such measurements could he highly informative, as has been shown by optical measurements [17, 18]. A difficulty is the need to analyze a multipath signal. Waveguide propagation in the atmosphere boundary layer requires special examination.

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