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MEASUREMENTS OF THE ATMOSPHERIC REFRACTION USING DIFFERENT RAY PATHS

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We obtain integral expressions relating the values of the astronomical refraction measured from the ground at a positive elevation angle, the limb refraction measured from a satellite using occultation technique at negative elevation angles, and the refraction measured by the immersion method in which a source or a receiver change their locations in the atmosphere.

1 . INTRODUCTION

At present, much attention is focused on the capabilities of remote sensing of the atmosphere, such as reconstruction of the altitude profile of the refractive index and other atmospheric parameters using the measured characteristics of radiation from extraterrestrial sources. Techniques aimed at reconstruction of the refractive-index profile are developed for reception schemes of different geometrical configuration, e.g., satellite-borne limb measurements of the refraction [1-22], ground-based measurements of the astronomical refraction [23-30], or refraction determination by the immersion technique [31, 32]. Some of the problems mentioned are ill-posed and can be solved if additional a priori information on the form of their solutions is used. It is interesting to compare the data obtained using different experimental schemes, since modern navigation systems allow one to perform simultaneous measurements over a wide variety of different ray paths. Of course, each scheme has its own advantages and drawbacks. For example, limb measurements have good resolution over altitude (geocentric distance), and the refraction is formed on a horizontal scale of about 1000km. Ground-based measurements have low altitude resolution but better binding to the measurement location. Measurements using the immersion method can yield good resolution in both vertical and horizontal directions. Each method has its own specific errors. In particular, the contributions of horizontal irregularities of the refraction index to refraction measurements is different, and this contribution should be treated as a measurement error if the altitude profiles are reconstructed in the framework of the spherically symmetric model of the atmosphere.

To adequately compare the results of refractometric methods, one should know the accuracy of matching of refraction measurements over different ray paths. This paper is aimed at obtaining formulas relating the values of refraction corresponding to different measurement schemes and allows one to calculate certain refraction value using the data obtained by another method. In this case, one should take into account the indicated discrepancies in refractometric data used to remote diagnosis of the altitude distributions of the atmospheric parameters.

2. INITIAL RELATIONS

To achieve the goals formulated above, we use initial integral relationships for the refraction angle measured using the corresponding ray paths. We use also the relevant formulas for their inversion with respect to the altitude profile of the refractive index. The latter formulas are obtained when solving inverse problems.

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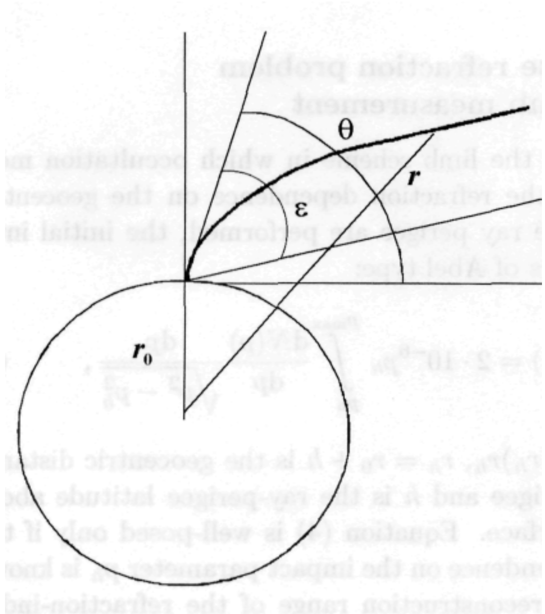

Fig. 1

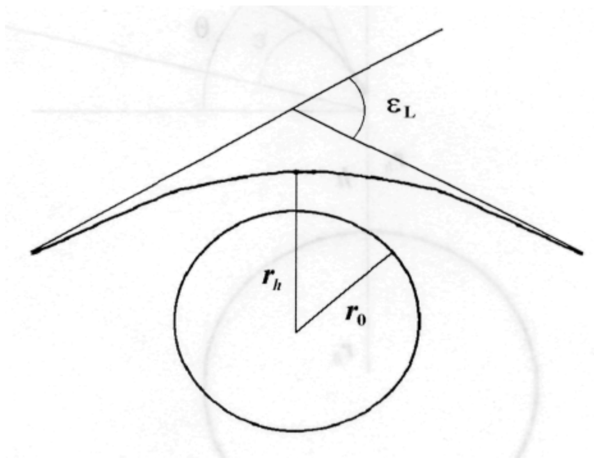

Fig. 2

2.1. Inverse problem for astronomical refraction

In this case, the following Fredholm integral equation of the 1st kind for the angle *e* of astronomical refraction as a function of the position angle θ of the incoming ray plays the role of the initial equation:

$$\varepsilon(p_\theta) = -10^{-6} p_\theta \int_{p_0}^{p_{\max}} \frac{\mathrm{d}N(p)}{\mathrm{d}p} \frac{\mathrm{d}p}{\sqrt{p^2 - p_\theta^2}}, \tag{1}$$

where $p = n(r)r$, n is the refractive index of the atmosphere, $r = r_0 + h$ is the geocentric distance, h is the altitude above the Earth's surface, $p_0 = n(r_0)r_0$, r_0 is the radius of the Earth, $p\theta = n(r_0)r_0\cos\theta$, and $N = 10^6$ $(n — 1)$ is the refraction index. A profile $N(p)$ *is* transformed into $N(h)$ using the relation

$$h = p/[1 + 10^{-6} N(p)] - r_0.$$

The geometrical scheme of the measurements is given in Fig. 1.

If the integration by parts is performed in initial relationship (1) for the astronomical refraction, this formula is reduced to

$$\tilde{\varepsilon}(p_\theta) = \int_{p_0}^{p_{\max}} N(p) \frac{p p_\theta \,\mathrm{d}p}{(p^2 - p_\theta^2)^{3/2}}, \tag{2}$$

where

$$\tilde{\varepsilon}(p_\theta) = -10^6 \varepsilon(p_\theta) + p_\theta \frac{N(p_0)}{\sqrt{p_0^2 - p_\theta^2}}, \tag{3}$$

so the value $N_0 = N(p_0)$ of the refraction index in the boundary layer becomes salient.

It is impossible to solve integral Fredholm equation (2) and find the explicit inversion formula for the inverse problem on reconstruction of refraction-index altitude profile. It is known that such a problem is ill-posed and should be solved using various regularization methods [23-30].

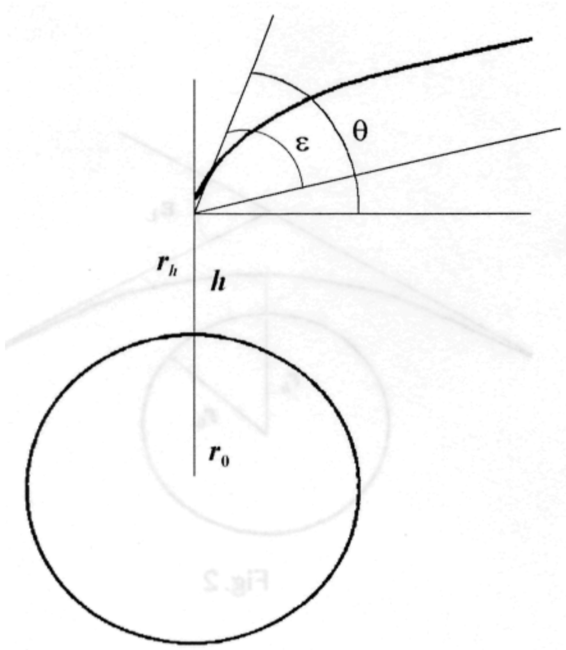

Fig. 3

2.2. Inverse refraction problem for limb measurement

In the case of the limb scheme in which occultation measurements of the refraction dependence on the geocentric distance of the ray perigee are performed, the initial integral equation is of Abel type:

$$\varepsilon_{\mathrm{L}}(p_h) = 2 \cdot 10^{-6} p_h \int_{p_h}^{p_{\max}} \frac{\mathrm{d}N(p)}{\mathrm{d}p} \frac{\mathrm{d}p}{\sqrt{p^2 - p_h^2}}, \tag{4}$$

where , $p_h = n(r_h)r_h$, $r_h = r_0 + h$ is the geocentric distance to the ray perigee and h is the ray-perigee latitude above the earth's surface. Equation (4) is well-posed only if the refraction dependence on the impact parameter p_h is known on the entire reconstruction range of the refraction-index profile. In this case, the inverse Abel transform [1-22]

$$N(p) = 10^6 \int_{p}^{p_{\max}} \varepsilon_{\mathrm{L}}(p_h) \frac{\mathrm{d}p_h}{\pi \sqrt{p_h^2 - p^2}} \tag{5}$$

yields the exact solution of Eq. (4) for the refractive-index profile. Figure 2 shows the geometric scheme of limb measurements.

2.3. Inverse refraction problem for immersion-method measurement

The inverse refraction problem for immersion-method measurements, in which the receiver or transmitter changes position in the studied atmosphere and the height dependence of refraction is measured, is reduced to reconstruction of the refraction-index profile from the equation [31-32]

$$\varepsilon(p_h) = -10^{-6} \int_{p_h}^{p_{\max}} \frac{\mathrm{d}N(p)}{\mathrm{d}p} \frac{p_0 \cos\theta \, \mathrm{d}p}{\sqrt{p^2 - p_0^2 \cos^2\theta(p_h)}}, \tag{6}$$

in which $p > p_h$. If the position angle θ of the incoming ray is constant, we have the following inversion formula for this equation [31]:

$$N(p) = -10^6 \int_{p}^{p_{\max}} \frac{\mathrm{d}\varepsilon(p_h)}{\mathrm{d}p_h} \frac{\sqrt{p_h^2 - p_0^2 \cos^2\theta(p_h)}}{p_0 \cos\theta} \mathrm{d}p_h. \tag{7}$$

The geometrical scheme of measurements by the immersion method is given in Fig. 3.

3. RELATIONSHIP BETWEEN THE REFRACTION ANGLES DETERMINED USING DIFFERENT RAY PATHS

We consider the problem on the determination of the relationship between the values of the astronomical refraction and the refraction obtained by limb measurements. This relationship can be obtained using Eqs. (1) and (5). Integrating by parts the refraction-index profile (5):

$$N(p) = -\frac{10^6}{\pi} \int_p^{p_{\max}} \frac{d\varepsilon(p_h)}{dp_h} \operatorname{arch} \frac{p_h}{p} \, dp_h,$$

substituting the result into the astronomical-refraction equation (1), and changing the order of integration, we arrive at the expression

$$\varepsilon(p_\theta) = -\frac{p_\theta}{2\pi} \int_{p_0}^{p_{\max}} \frac{d\varepsilon(p_h)}{dp_h} p_h \, dp_h \int_{p_0}^{p_h} \frac{dp}{p\sqrt{p_h^2 - p^2}\sqrt{p^2 - p_\theta^2}},$$

in which the inner integral can be evaluated in explicit form. As a result, we find the desired relationship between the astronomical refraction ε and the refraction ε_L determined by limb measurements:

$$\varepsilon(p_\theta) = -\frac{1}{2\pi} \int_{p_0}^{p_{\max}} \frac{d\varepsilon_L(p_h)}{dp_h} \left(\frac{\pi}{2} - \arcsin \frac{(p_h^2 + p_\theta^2)\, p_0^2 - 2p_h^2 p_\theta^2}{p_0^2 (p_h^2 - p_\theta^2)} \right) dp_h. \tag{8}$$

Integrating by parts, we present Eq. (8) in the following form:

$$\varepsilon(p_\theta) = \frac{p_\theta}{\pi} \int_{p_0}^{p_{\max}} \varepsilon_L(p_h) \sqrt{\frac{p_0^2 - p_\theta^2}{p_h^2 - p_0^2}} \frac{dp_h}{p_h^2 - p_\theta^2}, \tag{9}$$

where $p_0 \le p_h \le p_{\max}$.

In particular, in the limiting case θ = 0, Eq. (9) is reduced to the obvious relationship

$$\varepsilon(p_\theta \,|\, \theta = 0) = \varepsilon(p_0) = \frac{\varepsilon_L(p_0)}{2}. \tag{10}$$

If Eq. (2), in which the refraction-index altitude profile (5) is substituted, is integrated by parts and used instead of the initial equation (1) for astronomical refraction, we can find the formula relating astronomical and limb refraction:

$$\varepsilon(p_\theta) = 10^{-6} \left(p_\theta \frac{N(p_0)}{\sqrt{p_0^2 - p_\theta^2}} - \frac{10^6 p_\theta}{\pi} \int_{p_0}^{p_{\max}} \varepsilon_L(p_h) \sqrt{\frac{p_h^2 - p_0^2}{p_0^2 - p_\theta^2}} \frac{dp_h}{p_h^2 - p_\theta^2} \right), \tag{11}$$

in which the term determined by the ground value of refraction index is separated.

It is seen from Eqs. (9) and (11) that determination of the astronomical refraction, corresponding to ground-based measurements, using limb refractometric data is a well posed problem, while the inverse problem of determination of the limb refraction using position-angle measurements of the astronomical refraction is ill-posed and reduces to an integral Fredholm equation of the 1st kind.

Next, we consider the problem of determination of the relationship between the refraction measured by the immersion method and by limb measurements. To obtain the desired equation for the refraction angles ε and ε_L the refraction-index profile from inversion formula (7). Substituting this profile into Eq. (4), which corresponds to limb measurements, we get

$$\varepsilon_L(p_h) = 2p_h \int_{p_h}^{p_{\max}} \frac{d\varepsilon(p)}{dp} \frac{\sqrt{p^2 - p_0^2 \cos^2\theta}}{p_0 \cos\theta} \frac{dp}{\sqrt{p^2 - p_h^2}}. \tag{12}$$

Equation (12) allows one to determine the limb refraction in the case where the refraction is measured in the course of immersion of the source or receiver into the atmosphere. We should point out, however, that this problem is ill posed, since it contains a derivative with respect to the experimental data. Inversion of this equation requires the use of regularization methods. In this case, it is important that, integrating Eq. (12) by parts, one cannot eliminate the derivative in this integral. It is easily seen that the inverse problem on determination of the refraction by the immersion method using the limb measurements is reduced to Eq. (9) written for an arbitrary altitude inside the atmosphere. As was mentioned above, such a problem is well posed.

In principle, similar relations can be obtained for other characteristics of atmospheric emission and absorption measured using different ray paths.

4. CONCLUSION

In this paper, we consider problems on the relationships of refraction values determined using geometrically different experimental schemes, in particular, astronomical refraction determined from ground-based measurements at positive position angles, limb refraction corresponding to measurements at negative position angles, and refraction measured by the immersion method. We find the corresponding equations relating the astronomical and limb refraction, as well as the limb and immersion-method refraction. The obtained relationships can be used for comparison of the refraction data obtained using different ray paths and for evaluation of certain data using the data obtained in different measurements. Similar relationships can also be obtained for other parameters of the atmospheric emission and absorption.

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REFERENCES

1. A. J. Kliore, D.L. Gain, G.S.Levy, and V. R.Eshelman, *Astronaut. Aeronaut,* No. T-7, 72 (1965).
2. J.R. Lions and D.L. Sweetnam, V. R. Eshleman, et.al., *J. Geophys. Res.,* **92**, No. 13, 14987 (1987).
3. G. Fjedlbo and V. R. Eshleman, *Radio Sci.,* **4**, No. 10, 879 (1969).
4. A. J. Klior, G. Fjedlbo, and B. Seidel, *Radio Sci.,* **5**, No. 2, 373 (1970).
5. A. J. Kliore, J.R. Patel, B. Seidel, et.al., *J. Geophys. Res.,* **85**, No, A-ll, 5857 (1980).
6. G. F. Lindal, D. L. Sweetnam, and V. R. Eshleman, *Astron. J,.* **90**, No. 6, 1136 (1985).
7. G.F. Lindal, J.R. Lions, D.L. Sweetnam, et.al., *J. Geophys. Res.,* **92**, No. A-13, 14987 (1987).
8. G.F. Lindal, J.R. Lions, D.L. Sweetnam, et al., J. Geophys. Res., **17**, No. 10, 1733 (1990).
9. M.A. Kolosov, O.I.Yakovlev, Yu.I. Kruglov, et.al., *Radiotekh. Elektron.,* **17**, No. 12, 2483 (1972).
10. G. M. Grechko, A. S. Gurvich, Yu V. Romanenko, S. A. Savchenko, and S. V. Sokolovsky, *Dokl. Akad. Nauk SSSR,* No. 4, 828 (1979).
11. S. V. Sokolovsky, *bv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **17**, No. 6, 574 (1981).
12. A. S. Gurvich, V. Kan, L. I. Popov, V. V. Ryumin, and S. A. Savchenko, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **18**, No. 1, 3 (1982).
13. K. P. Gaikovich and A. P. Naumov, *Issled. Zemli iz Kosmosa,* No. 4, 25 (1983).
14. K.P. Gaikovich, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **20**, No. 8, 675 (1984).
15. S. V. Sokolovsky, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **22**, No. 8, 890 (1986).
16. A. A. Volkov, G. M. Grechko, A. S. Gurvich, et.al., *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **23**, No. 11, 1228 (1987).
17. G. M. Grechko, A. S. Gurvich, V. A. Kazbanov, et. al., *Trudy GOI,* **71**, No. 205 (1989).

18. S. P. Beschastnov, G. M. Grechko, A. S. Gurvich, et. al., *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **20**, No. 4, 231 (1984).

19. M. E. Gorbunov, *Izv. Akad. Nauk SSSR, Scr. Fiz. Atmos. Okeana,* **30**, No. 6, 776 (1994).

20. K.P. Gaikovich, A. S. Gurvich, and A. P.Naumov, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **19**, No. 7, 675 (1983).

21. S. P. Beschastnov, G. M. Grechko, A. S. Gurvich, et. al., *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **20**, No. 2. .231 (1984).

22. S. V. Zagoruyko and V. Kan, *Radiotekh. Elektron.,* **29**, No. 5, 95 (1984).

23. A. G. Pavelyev, *Radiotekh. Elektron.,* **25**, No. 12, 2504 (1980).

24. A. G. Pavelyev, *Radiotekh. Elektron.,* **27**, No. 5, 1037 (1982).

25. M. A. Kolosov and A. G. Pavelyev, *Radiotekh. Elektron.,* **27**, No. 12, 2310 (1982).

26. A. G. Pavelyev, *Zh. Vychisl. Mat. Mat. Fiz.,* **25**, No. 3, 392 (1985).

27. K.P. Gaikovich and M.I.Sumin, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **22**, No. 9, 917 (1986).

28. N. A. Vasilenko, K.P. Gaikovich, and M.I.Sumin, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana,* **22**, No. 10, 1026 (1986).

29. N. A. Vasilenko, K.P. Gaikovich, and M.I.Sumin, *Dokl. Akad. Nauk SSSR,* **290**, No. 6, 1332 (1986).

30. N. A. Vasilenko, K.P. Gaikovich, and M. B. Chernyaeva, *Izv. Vyssh. Uchebn. Zaved., Radiofiz.,* **40**, No. 6, 682 (1997).

31. K.P. Gaikovich and G.Yu. Khacheva, *Opt. Atmos. Okeana,* **10**, No. 1, 69 (1997).

32. K.P. Gaikovich and G.Yu. Khacheva, in: Con/. *Proc. of 7th: Int. Crimean Conf. "Microwave and Telecommunication Technology" (Crimea, Ukraine, Sevastopol, Sept. 15-18, 1997),* Weber Co., Sev astopol (1997), p. 681.