DETERMINATION OF HEAT SOURCES FROM THE THERMAL RADIATION OF A HALF SPACE WITH STEADY-STATE TEMPERATURE DISTRIBUTION

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An equation is obtained for the brightness temperature of the thermal radiation from a half space, expressing the dependence of that quantity upon the depth distribution of thermal sources. It is shown that such an equation can be used to reconstruct both thermal source distributions and subsurface temperature profiles. The capabilities of the approach are demonstrated by reconstruction of parameters of a frozen soil layer from radiothermal radiation emitted.

The development of radiometry means and methods has created ever greater capabilities for subsurface probing of various media (soils, water, biological media) by measuring their radiothermal radiation spectra. Such problems were considered in [1-7]. In a number of situations an approach based on simultaneous solution of the radiation transport and thermal conductivity equations have proved fruitful, allowing derivation of thermoevolution equations [5,7], which establish direct relationships between evolution of boundary conditions on the surface (temperature or thermal flux) and the dynamics of observed brightness temperatures. By using the time dependence it proves possible to formulate and solve a number of problems related to reconstruction of underground profiles, thermal history, and determination of parameters of the radiating half-space [5,7].

In the present study we will consider another, yet similar case, where the temperature distribution in the half space is steady state and corresponds to thermal sources distributed over depth. Such problems arise in probing biological tissues, upon presence in the medium of thermal sources of a different nature, for example, regions in which phase transitions occur.

1. Theoretical Analysis. We will consider the model of a plane half-space $z \leq 0$ with absorption coefficient γ and thermal conductivity k. Let the thermal source function $W(z)$ depend solely on depth. In this case the one-dimensional

$$k(d^2T/dz^2) = W(z). \tag{1}$$

The thermal radiation brightness temperature obtained by measurements into the depths satisfies the well-known relationship

$$T_B(\lambda) = (1 - R) \int_{-\infty}^{0} T(z)\ \gamma(\lambda)\ e^{\gamma z}\ dz. \tag{2}$$

where R is the reflection coefficient at the boundary of the half space (for simplicity, below we will assume that $R = 0$).

Integrating by parts twice in Eq.(2), we have

$$T_B(\lambda) = T(0) - \frac{1}{\gamma} \frac{dT}{dz}(0) + \int_{-\infty}^{0} \frac{d^2T}{dz^2} \frac{1}{\gamma} e^{\gamma z}\ dz. \tag{3}$$

Substituting Eq.(1) in Eq.(3), we obtain the expression

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$$T_B(\lambda) = T(0) - \frac{1}{\gamma} \frac{dT}{dz}(0) + \frac{1}{\gamma k} \int_{-\infty}^{0} W(z)\, e^{\gamma z}\, dz. \tag{4}$$

Considering further that the heat can move only through the upper boundary, for the thermal flux J through the surface z = 0 we obtain

$$J = k \frac{dT}{dz}(0) = \int_{-\infty}^{0} W(z)\, dz. \tag{5}$$

Substituting Eq.(5)in Eq.(4), we then have the desired relationship

$$T_B(\lambda) = T(0) + \frac{1}{\gamma k} \int_{-\infty}^{0} W(z)\, (e^{\gamma z} - 1)\, dz. \tag{6}$$

If we use another possibility for choosing the limits for the integration by parts, we easily obtain an analog of Eq.(6), but in place of $T(0)$ there appears the temperature of the deep layers $T(-\infty)$, which in a number of cases proves more useful useful:

$$T_B(\lambda) = T(-\infty) + \frac{1}{k} \int_{-\infty}^{0} W(z) \left[\frac{1}{\gamma}(e^{\gamma z} - 1) - z\right] dz. \tag{7}$$

Equations (6), (7) are Fredholm integral equations of the first kind relative to the source function $W(z)$ (the surface temperature can be measured by contact methods or in the IR range). As is well known [7], solution of this equation is an incorrect problem and requires use of adequate a priori information on the function $W(z)$. On the basis of the reconstructed $W(z)$ distribution by integrating Eq.(1) we can reconstruct the subsurface temperature profile $T(z)$. The natural question arises of what the advantage of the analogous problem of Eq.(2) directly for $T(z)$, as was done, for example, in [4]. The point is that *a priori* information on $W(z)$, in particular, on localization of thermal sources, may be more accessible and more convenient in form that similar data for $T(z)$, especially when the sources occupy a relatively small depth interval. Just such a situation exists when frozen ground is probed and the heat sources are localized in the phase transition zone. The depth of the transition zone varies very slowly, so that the problem is quasi-stationary.

2. Determination of Frozen Soil Parameters from Radiometric Data. In February and March 1987 measurements of the thermal radiation of a clay —sand soil were performed at wavelengths of 3, 9, and 13 cm. In the measurements the antenna system was located underneath a plane metallic screen to compensate reflection (R = 0). Contact measurements of $T(z)$were performed simultaneously. Over the measurement period the depth of the frozen layer varied from 95 to 105 cm. The results of retrieval of $T(z)$ on the basis of Eq.(2) and estimates of the soil parameters were published by the present authors in [5,6].

We will consider the possibility of analyzing the data obtained on the basis of Eq.(6). We will neglect the thickness and motion of the phase transition zone, so that the thermal source function can be represented in the form

$$W(z) = \Lambda \delta(z^{*}). \tag{8}$$

Substituting Eq.(8)in Eq.(6), we have

$$T_B = T(0) + (\Lambda/\gamma k)(e^{\gamma z} - 1). \tag{9}$$

Substituting Eq.(8)in Eq.(7), we can obtain an analogous expression for the case where the value of $T(-\infty)$ is known.

From Eq.(9) we can define the thermal source power in the phase transition zone

$$\Lambda = \frac{(T_B - T(0))\gamma k}{e^{\gamma z^{*}} - 1} \tag{10}$$

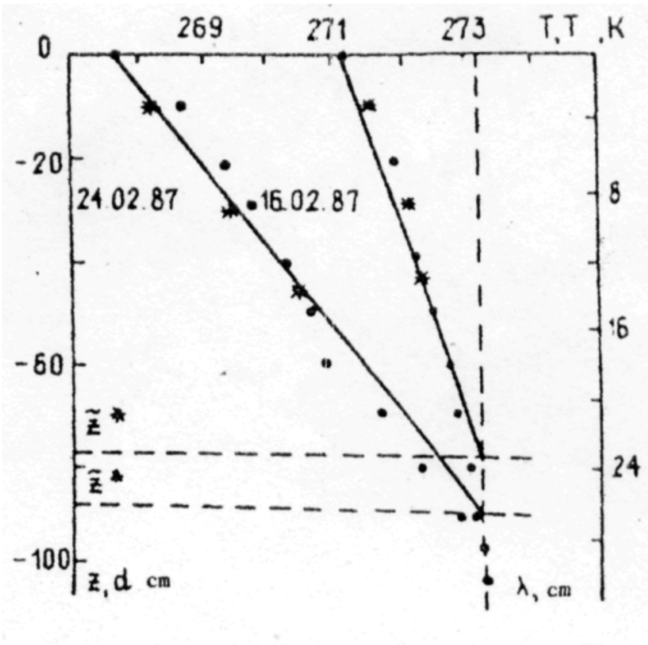

Fig. 1

or, choosing the wavelength sufficiently small, so that $(-\gamma z^*) \gg 1$,

$$\Lambda = \gamma k \;(T(0) - T_B). \tag{11}$$

From Eq.(11) we find the temperature gradient in the frozen layer

$$\frac{dT}{dz} = \begin{cases} \Lambda/k, & z \geq z^* \\ 0, & z < z^* \end{cases} \tag{12}$$

and temperature profile

$$T(z) = \begin{cases} T(0) - (\Lambda/k)z, & z \geq z^* \\ 273 \text{ K}, & z < z^* \end{cases}. \tag{13}$$

From Eqs.(11) and (13) we find the freezing depth

$$z^* = (T(0) - 273,15)\; k/\Lambda. \tag{14}$$

Figure 1 shows results from [6] for measurement of frozen ground on two observation days (points, contact temperature measurements vs depth; stars, soil brightness temperature at wavelengths λ = 3; 9, and 13 cm, shown as function of skin-layer thickness $d = 1/\gamma(\lambda)$; solid lines, $T(z)$ profiles from Eq.(13), on the basis of which z was estimated according to Eq.(14)). With consideration of soil parameters [6, 81] for dry ground we obtain the estimate $k = 2 \cdot 10^{-3}$ cal/cm·deg·sec. On the basis of the data obtained in [6] we have $\gamma \approx 31/\lambda$ cm^{-1}. From Eq.(11) we use the T_B values (λ = 13 cm) and $T(0)$ (see Fig.1) to define the thermal source power in the phase transition zone

$$\Lambda = \begin{cases} -4,7 - 10^{-5} \text{ cal/cm}^2\cdot\text{sec} & (16.02.87) \\ -1,2 - 10^{-4} & (24.02.87) \end{cases}$$

while we use Eq. (14) to determine the freezing depth

$$z^{*} = \begin{cases} 79 \text{ cm} & (16.02.87) \\ 89 \text{ cm} & (24.02.87) \end{cases} .$$

Considering that there was a gradual temperature decrease from the 16th to the 24th, we take as the mean heat liberation value for freezing of water $A = 8 \cdot 10^{-6}$ cal/cm$^2 \cdot$sec. Then, through a 1 cm^2 section over 8h there is liberated a quantity of heat $Q \approx 52$ cal, so that a mass of liquid water $m = 0.65$ g freezes. Over this time period the freezing depth increases by approximately 10 cm, which permits the estimate of volume moisture content in the soil $W_{H2O} = 6.5\%$.

We will note the role of snow coating in the measurements: on the one hand, the dry snow cover is transparent in the cm range and does not hinder measurements of soil radiation, while on the other, due to its low thermal conductivity it stabilizes the surface temperature, which insures applicability of the condition of stationary $T(z)$ with good accuracy.

Thus, in cases where the heat source is localized in some plane surface, Eqs.(6), (7) have an exact solution. In the case of distributed sources the solution can be found by methods developed in [3,4,6] for solution of the analogous inhomogeneous problem of temperature profile reconstruction.

On the basis of the equations obtained, which express the dependence of half-space brightness temperature on thermal source distribution over depth, an analysis has been performed of radio radiation measurements from frozen ground. Estimates were obtained of the freezing depth and rate of latent heat liberation in the phase transition zone, while data on the mean rate of increase in thickness of the frozen layer was used to determine moisture content.

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