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ANALYSIS OF SOLUTION ACCURACY FOR DIRECT AND INVERSE PROBLEMS OF ASTRONOMICAL REFRACTION USING EXPERIMENTAL DATA

N. A. Vasilenko, K. P. Gaikovich,
and M. B. Chernyaeva

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In this paper we analyze the actually attainable accuracy of refractive index reconstruction in the optical range from the solution of the inverse problem of astronomical refraction, processing a large set of data of simultaneous astronomical and aerological measurements. Two main results are obtained. First, standard deviations of the measured values of astronomical refraction from those calculated from the sounding data are determined, which allows us to estimate the effective error in refraction measurement and analyze its components. Second, comparing the reconstructed altitude profiles of atmospheric parameters with the sounding data, we estimate the maximum errors in the profile reconstruction and analyze their structure.

1. INTRODUCTION

A number of papers have presented the possibility of using the measurements of astronomical refraction of space sources for reconstruction of the altitude profile of the refractive index and the related equations of statics and gas state of such parameters as pressure and temperature [1-5]. This problem is of interest in view of the perspective use of the measurements of refraction of signals from navigation satellites in the radiowave spectrum [5] to carry out constant monitoring of the refractive index profile, which, in particular, would allow us to promptly predict the on-path propagation conditions. It is of interest to estimate the maximum potential of the method in the optical range in which measurements are not always possible because of the clouds covering the sources.

If, to reconstruct the refractive index profile, we use the measurements of astronomical refraction, i.e., the ground-based measurements of refraction at positive angles of elevation, the problem is reduced to solving a Fredholm integral equation of the 1st kind and is incorrect mathematically [4].

The range of the angles of elevation, which are informative for measurements, is determined in [1-3]. This range is bounded from above by the angles at which refraction variations become comparable with measurement errors, which results from the "Laplace theorem" [6]. With the measurement accuracy $\delta\varepsilon = 1''$ the range of informative angles is $0 < \theta_0 < 4°$ and $0 < \theta_0 < 2°$ for $\delta\varepsilon = 10''$.

The numerical solution of the inverse problem of astronomical refraction for positive angles of elevation requires a priori information, without which a solution is impossible, generally speaking [1-3].

In this case we used the method of statistical regularization [1-3, 5] in which the information on the interlevel covariance ties of the refractive index is used directly. The solution is sought at the a priori ensemble specified by the covariance matrix.

Since the reconstruction accuracy depends on the class in which the solution is sought, on the solution method, and on additional constraints, and since an increase in the reconstruction accuracy for an incorrect problem is much slower than a proportional increase, to estimate the error, a closed-loop numerical experiment was conducted in [1-3].

Numerical experiments show that the reconstruction accuracy is not the same for different realizations of the altitude profile of the given refractive index for the same measurement accuracy. Therefore, individual reconstruction examples fail to estimate objectively the solution error for the incorrect problem under

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consideration. Therefore, to obtain reliable data on the reconstruction accuracy, the reconstructed profiles of the refractive index, pressure, and temperature are studied statistically in [1] over large ensembles of meteorological information.

However, the theoretical study of the possibilities of reconstruction of atmospheric parameters from the refraction measurements does not solve a number of important problems related to the practical use of the method, which can be solved only experimentally. First, this is a problem of the role of horizontal inhomogeneities of the refractive index in the actual atmosphere, which introduce an additional error which does not lend itself to easy estimation within the framework of the theoretical analysis. Also, without experiment it is difficult to allow for the errors introduced to refraction by the turbulent fluctuations of the refractive index and for the errors related to its certain dispersion in the optical range. In [2, 3] the atmospheric parameter profiles are reconstructed for characteristic cases (monotonic profiles, near-earth and elevated inversions). However, the great volume of the experimental data available in the optical range, which are accompanied by the simultaneous aerosounding measurements of atmospheric parameter profiles at the same place (the measurement conditions and methods are described in detail in [2, 3]), allows us to carry out complete statistical analysis of the really attainable accuracy of the method. First, it is possible to compare statistically the measured refraction values with those calculated from the sounding data. Second, comparing the reconstructed profile of the refractive index with the profile reconstructed from the sounding data, we can estimate the profile reconstruction accuracy.

2. COMPARISON OF MEASURED REFRACTION VALUES WITH THE RESULTS OF SOUNDING-BASED CALCULATIONS

For the angle of astronomical refraction we write the equation [3]

$$\int_{p_0}^{p_H} N(p)\frac{p p_\theta dp}{(p^2 - p_\theta^2)^{3/2}} = \tilde{\varepsilon}\,(p_\theta), \tag{1}$$

where

$$\tilde{\varepsilon}\,(p_\theta) = -10^{-6}\varepsilon(p_\theta) + p_\theta\left(\frac{N(p_0)}{\sqrt{p_0^2 - p_\theta^2}} - \frac{N(p_H)}{\sqrt{p_H^2 - p_\theta^2}}\right) \tag{2}$$

$N = 10^6(n - 1)$, $p = nr$, $p\theta$=n0r0cosθ0, $p_0 = n_0 r_0$, $n_0 = n(r_0)$ is the refraction angle, θ_0 is the angle of elevation of the beam arrival, and n is the refractive index.

The results of statistical analysis show that the standard deviation of refraction calculated from the sounding data from the refraction measured at angles of elevation smaller than 5° is 12" for an estimated refraction measurement accuracy of 6". The refraction calculation accuracy is influenced by the experimental data errors due to the sounding balloon inertia and the influence of horizontal inhomogeneities on both the readings from a wind-driven sounding balloon and refraction itself. Assuming that the refraction measurement error and the error due to the above inaccuracies of the sounding results are not related to each other, we obtain $(\delta\varepsilon)^2 = \sigma^2_{sound} + \sigma^2_{meas}$ where $\delta\varepsilon$ is the maximum accuracy of refraction determination, σ_{sound} is the error due to the sounding inaccuracies, and σ_{meas} is the refraction measurement error from which we find σ_{sound}. For $\delta\varepsilon$ = 12" and σ_{meas} = 6" this error is 10.4".

3. STATISTICAL ANALYSIS OF THE RECONSTRUCTED PROFILES OF THE REFRACTIVE INDEX

The numerical simulation results [2] show that for errors of ~ 12" the accuracy of the refractive index reconstruction from the solution of the inverse problem of astronomical refraction, which is reduced to the solution of Eq. (1), is much better than the accuracy of reconstruction by the method of optimal extrapolation from the near-earth value:

$$N^{o}(h) = \langle N(h)\rangle + \frac{B_{NN}(0,h)}{\sigma_N^2(0,0)}\left(N_0 - \langle N_0\rangle\right). \tag{3}$$

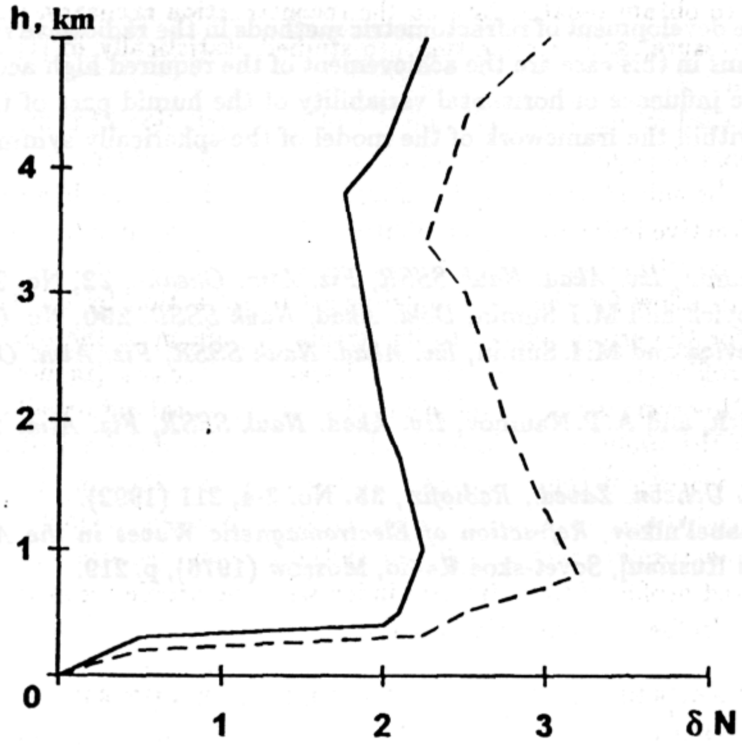

Fig. 1. Altitude dependence of the standard deviation of reconstructed refractive index from the sounding value (solid curve) compared with the accuracy of linear regression estimate of the refractive index profile from its near-ground value (dashed curve).

In this paper we obtain the altitude dependences of the standard deviations of the refractive index profiles reconstructed using the method of statistical regularization and optimal extrapolation on the sounding profiles for the above error level $\delta\varepsilon$ = 12". The covariance matrix used in the method of statistical regularization was calculated from the data from 30 sounding balloons on the pressure, humidity, and temperature, which were launched simultaneously and at the same place with astronomical refraction measurements. The calculated results are given in Fig. 1. It is evident that at an altitude of from 0 to 4 km the accuracy estimated by the method of statistical regularization is $1.75 < \delta N < 2.25$, whereas the method of optimal extrapolation gives $2.3 < \delta N^{ex} < 3.25$ for the same altitudes.

Processing of experimental data shows that the refraction measurement accuracy, the accuracy of algorithms for its calculation, as well as the adequacy of approximation of the spherically symmetric atmosphere are sufficient for qualitative reconstruction of the atmospheric parameter profiles, at least, for stable weather conditions in continental regions with a uniform underlying surface. Aerological data provide sufficiently accurate determination of refraction from calculations and can be used for comparison with reconstructed profiles. The reconstruction accuracy corresponds to theoretical estimates for the level of realized accuracy of measurements and calculated refraction values.

4. CONCLUSION

In this paper we have considered the problem of reconstruction of the altitude profile of the atmospheric refractive index from measurements of the astronomical refraction of space sources. The accuracy of refractive index reconstruction in the optical range from the solution of the inverse problem of astronomic refraction has been analyzed statistically on the basis of simultaneous astronomical and aerological measurements. The standard deviation of refraction calculated from the sounding data from the measured refraction, which is 12" for the known estimated accuracy of refraction measurements of 6", has been obtained. The refraction calculation error related to the inaccuracies of the sounding data was found to be 10.4". The standard deviation of the reconstructed profile *N(h)* from the sounding results is *2.5N* units in a layer of from 0 to 4 km.

The results show that the development of refractometric methods in the radiowave range [5] has certain perspectives. The main problems in this case are the achievement of the required high accuracy of refraction measurements and the possible influence of horizontal variability of the humid part of the refractive index in the case of reconstruction within the framework of the model of the spherically symmetric atmosphere.

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